الجمهورية الجزائرية الديمقراطية الشعبية
REPUBLIQUE ALGERIENNE DEMOCRATIQUE ET POPULAIRE
وزارة التعليم العالي والبحث العلمي
Ministère de l'Enseignement Supérieur et de la Recherche Scientifique

Université Djilali BOUNAAMA Khemis Miliana
Faculté des Sciences et de la Technologie
Conseil Scientifique de la Faculté

جامعة الجيلالي بونعامة خميس مليانة
كلية العلوم والتكنولوجيا
المجلس العلمي للكلية

Réf: 03/02/UNDBKM/FST/CSF/2026

EXTRAIT DU PV
DE LA REUNION EXTRA ORDINAIRE DU CONSEIL SCIENTIFIQUE DE LA FACULTE
DU 28/04/2026

L'an deux mille vingt-six (2026), le vingt-huit (28) avril, une réunion extraordinaire du Conseil scientifique de la Faculté des sciences et de la technologie s'est tenue à 10 h 00 dans la salle de réunion de la faculté.

DEROULEMENT

Après une allocution de bienvenue prononcée par le Président du Conseil scientifique, et en présence des membres, la séance a été déclarée ouverte. Les points inscrits à l'ordre du jour ont ensuite été examinés et débattus comme suit :

Objet : EXPERTISE DES POLYCOPIES PEDAGOGIQUES

Le Conseil Scientifique de la Faculté a validé la conformité des polycopiés pédagogiques présentés par les enseignants listés ci-dessous, sur la base des rapports favorables émis par les experts, comme détaillé ci-après :

Auteur du polycopié: BENTCHIKOU IBRAHIM (MCB), Département de Génie Electrique

Intitulé du polycopié	Linear Continuous Feedback Control Systems **(Rédigé en anglais)**	
Destiné aux étudiants	L2Automatique	
Experts du polycopié :		
YOUSFIAbdelkader	Pr	Univ-Khemis Miliana
TLEMCANI Abdelhalim	Pr	Univ-Médéa

L'ordre du jour achevé, la séance fut levée à 12h30.

Président du CSF
Pr CHAOUCHI Belkacem

رئيس المجلس العلمي
أ.د شاوشي بلقاسم

الجمهورية الجزائرية الديمقراطية الشعبية

Democratic and Popular Republic of Algeria

وزارة التعليم العالي والبحث العلمي

Ministry of Higher Education and Scientific Research

Djilali Bounaama Khemis Miliana University جامعة الجيلالي بونعامة خميس مليانة
Faculty of Science and Technology كلية العلوم والتكنولوجيا
Department of Technology قسم التكنولوجيا

Adresse : Rue Thniet El Had, Khemis Miliana, Ain Defla , Algérie. Tel :(213) 27556844

Title of the course handout:

Linear Continuous Feedback Control Systems

Intended for students:

Level : Second year licence

Speciality **:** Automatic control engineering.

Author

Bentchikou Ibrahim

Handout experts	**Grade**	**Affiliation establishment**
Tlemçani Abdelhalim	Full Professor	Médéa University
Abdelkader YOUSFI	MCA.	Khemis Miliana University

Date of validation of the handout by the authorized scientific body SCD and/or SCF:

SCD	……………….
SCF	……………….

Academic year : 2025/2026.

Semestre: 4
Unité d'enseignement: UEF 2.2.1
Matière 1: Systèmes asservis linéaires et continus
VHS: 67h30 (Cours: 3h00, TD: 1h30)
Crédits: 6
Coefficient: 3

Objectifs de l'enseignement:
Ce cours permettra à l'étudiant d'acquérir des connaissances sur la théorie de la commande des systèmes linéaires continus ainsi que sur les méthodes de représentation et d'analyse. A la fin du cours, les étudiants seront capables de modéliser, d'analyser et de concevoir des contrôleurs simples pour les systèmes automatisés.

Connaissances préalables recommandées
- Mathématiques de base (Algèbre, analyse, notamment la manipulation des valeurs complexes, …)
- Notions fondamentales d'électronique de base (circuits linéaires) et de physique.

Contenu de la matière :

Chapitre 1 : Généralités sur les systèmes asservis (2 Semaines)
Aperçu sur l'historique des systèmes de régulation, Terminologie des systèmes asservis (perturbation, consigne, commande, sortie, bruit de mesure, écart, poursuite, régulation, correcteur, …), Fonctions d'automatique (surveillances/sécurité, asservissement/régulation), Commande en boucle ouverte/ boucle fermée, Structure et organes d'un système de commande.

Chapitre 2 : Transformées de Laplace et Représentation des systèmes asservis (3 Semaines)
Transformée de Laplace des fonctions usuelles (définitions, propriétés, théorème de la valeur initiale et finale, …), Transformée de Laplace inverse (définitions, propriétés, …), Modèle mathématique d'un système, Représentation par les équations différentielles, Représentation des systèmes asservis par des fonctions de transfert (définition du gain statique, pôles, zéros d'une fonction de transfert), Schémas blocs et règles de simplification : systèmes séries, parallèles, à retour unitaire et non unitaire, …

Chapitre 3 : Analyse dans le domaine temporel (2 Semaines)
Régime transitoire, régime permanent et notions de stabilité, rapidité et précision statique, Notion de réponse impulsionnelle, Réponse des systèmes de premier et de second ordre pour des signaux typiques, Cas de systèmes d'ordre supérieur, Identification des systèmes de premier et de second ordre à partir de la réponse temporelle.

Chapitre 4 : Analyse des systèmes dans le domaine fréquentiel (4 Semaines)
Introduction, Représentation graphique des fonctions de transfert (diagrammes de Bode, lieu de Nyquist, abaques de Black-Nichols), Analyse et critères de stabilité (critère du revers dans le plan Bode/Nyquist, critère de Nyquist, lieu d'Evans, critère de Routh)

Chapitre 5 : Synthèse des systèmes (4 Semaines)
Introduction, Spécifications de synthèse (stabilité, rapidité, précision), Différentes structures

Foreword

This document is intended for second-year undergraduate students in Automatic Control, as well as for those in related fields (industrial systems, electronics, electrical engineering, computer science) and certain Master's programs (renewable energies, mechatronics, optimization of production systems). It complies with the official curriculum.

The objective is to provide students with a solid understanding of the fundamental principles of feedback control systems and regulation. This involves analyzing, modeling, and designing control laws for various technological systems.

This educational resource aims to facilitate the learning of essential concepts related to control and regulation. It offers a clear synthesis of the module's theoretical foundations for effective practical application.

Designed with a didactic approach, this document aspires to provide structured and coherent instruction, emphasizing fundamental ideas in an accessible and concise manner.

Here are the names and affiliations of the experts for the course handout:

Tlemçani Abdelhalim	Full Professor	Automatique	Médéa University
Abdelkader YOUSFI	MCA	Electrotechnique	Khemis Miliana University

Abstract

This course provides an in-depth analysis of fundamental control system architectures, from mathematical modeling to controller design and stability analysis. Structured into five key chapters:

- **System Modeling & Transfer Functions:** Laplace transform, block diagram reduction, and signal flow graphs.
- **Temporal & Frequency Analysis:** Step response, impulse response, stability criteria (Routh, Hurwitz), Bode and Nyquist diagrams.
- **Performance Specifications:** Accuracy (static error), speed (rise time, settling time), overshoot, and steady-state behavior.
- **Classical Controllers:** Proportional (P), Integral (I), Derivative (D), PID controllers, and lead/lag compensators.
- **Synthesis & Tuning Methods:** Root locus techniques, Ziegler-Nichols tuning, frequency-based compensator design.

Objectives:

- Master the analysis and design of closed-loop control systems.
- Acquire skills in stability assessment and performance optimization.
- Prepare for advanced specializations (robotics, embedded control, automation).

Keywords:
Servo control, Transfer function, Feedback loop, PID controller, Stability (Routh-Hurwitz, Nyquist), Root locus, Bode diagram, Temporal response, Static error, Lead-lag compensator, System modeling, Automatic regulation.

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General Introduction

The analysis and control of dynamic systems are fundamental pillars of modern engineering. From autonomous vehicles and robotic arms to precision temperature regulation and power grid management, linear control systems are the invisible architects of stability and performance. This course, "Linear Control Systems," is designed to provide students with a rigorous yet practical foundation in the theory and application of controlling linear, continuous-time dynamic systems. By bridging mathematical concepts with engineering intuition, this module equips students to model, analyze, and design simple yet effective controllers for automated systems.

0.1 Content and Schedule

The content of this course on Linear Control Systems is organized into five principal chapters, progressing from fundamental concepts to practical synthesis methods:

Chapter 1: General Introduction to Servo Systems

This introductory chapter establishes the essential terminology and architectural principles of control systems. We will examine in detail:

A brief history of regulation and control systems.

Core terminology (reference, disturbance, command, output, tracking error, controller, plant).

The key functions of automation: monitoring/safety vs. servoing/regulation.

A comparative analysis of open-loop and closed-loop control structures.

The functional components and architecture of a generic control system.

Chapter 2: Laplace Transforms and System Representation

This chapter provides the essential mathematical toolkit for modeling linear systems. Key topics include:

Definition, properties, and key theorems (Initial Value, Final Value) of the Laplace Transform.

The Inverse Laplace Transform and its application to differential equations From differential equations to transfer functions: definition, gain, poles, and zeros.

Block diagram algebra and simplification rules for series, parallel, and feedback (unity and non-unity) configurations.

Chapter 3: Time-Domain Analysis

This core analytical chapter focuses on system behavior as a function of time. We will cover:

Transient response, steady-state response, and the fundamental notions of stability, speed, and static precision.

The concept of impulse response and its significance.

Detailed analysis of first and second-order system responses to typical inputs (step, ramp).
Methods for identifying first and second-order systems from experimental time response data.

Chapter 4: Frequency-Domain Analysis

This applied module introduces powerful graphical tools for analyzing system stability and performance. Topics include:
Graphical representation of transfer functions: Bode diagrams, Nyquist plots, and Black-Nichols charts.
Stability analysis and criteria: the reversal criterion, Nyquist stability criterion, and the Routh-Hurwitz criterion.
Introduction to the root locus (Evans' method) for analyzing closed-loop pole movement.

Chapter 5: Control System Synthesis

The course culminates with the practical design of controllers. This chapter focuses on:
Defining synthesis specifications: stability, speed (response time), and steady-state accuracy.
An overview of standard compensator structures: lead/lag, PID, and RST controllers.
Criteria for selecting the appropriate compensator based on performance specifications.
Practical compensator tuning methods: Empirical methods (Ziegler-Nichols, Cohen-Coon, Symmetric optimum) and graphical synthesis methods using Root Locus, Bode, and Nyquist plots.

0.2 Learning Objectives

This program is designed to achieve three fundamental objectives:

1 **Acquisition of Technical Knowledge:**

- ✓ Master the specialized vocabulary of control systems engineering.
- ✓ Understand the mathematical foundations (Laplace transforms, complex analysis) for linear system representation.
- ✓ Assimilate the principles of time-domain and frequency-domain analysis.

2 **Development of Practical Skills:**

- ✓ Learn to model real-world physical systems (electrical, mechanical) using transfer functions.
- ✓ Master the analysis of system stability, speed, and precision using analytical and graphical methods.
- ✓ Be able to design and tune simple PID and lead/lag compensators for desired specifications.

3 **Critical and Forward-Looking Approach:**

- ✓ Develop the ability to compare and choose between different controller architectures (e.g., P vs. PI vs. PID).
- ✓ Understand the fundamental trade-offs in control design (e.g., speed vs. stability, performance vs. robustness).
- ✓ Build a foundation for advanced courses in digital control, robust control, and nonlinear systems.

4 Teaching Methodology

The pedagogical approach is built on three pillars:

4.1 **Lectures:** Structured presentations of theoretical concepts, illustrated with concrete case studies and solved examples. Each session utilizes detailed visual aids (block diagrams, Bode plots, root locus graphs) and precise bibliographic references.

4.2**Tutorials (Problem-Solving Sessions):** Hands-on sessions allowing students to apply theoretical concepts to concrete engineering problems. These sessions include:

- ✓ Reduction of complex block diagrams.
- ✓ Calculation of time-response characteristics (overshoot, settling time).
- ✓ Stability analysis using Routh-Hurwitz, Bode, and Nyquist criteria.
- ✓ Controller design exercises using Ziegler-Nichols tuning rules.

4.3 Laboratory Work (Practical Sessions): Hands-on manipulation using simulation software (e.g., MATLAB/Simulink, Scilab/Xcos) to concretize learning. Students will:

- ✓ Model and simulate first, second, and higher-order systems.
- ✓ Validate theoretical responses (step response, frequency response).

Implement and test PID controllers on simulated or physical prototypes, comparing performance against theoretical predictions.

Dr. Bentchikou Ibrahim

Chapter 1: General Concepts of Control Systems

1.1 Introduction :

Automation is the discipline that, in general, deals with the control of systems. It therefore plays a very important role in the industrial field to which It provides solutions, study methods, and systematic analytical approaches.

1.1 .1. Historical overview

Control systems have their origins in antiquity, notably with the float regulator for water clocks. However, it was in the 19th century that the first industrial applications appeared, such as Watt's centrifugal governor used in steam engines. Automation then evolved rapidly in the 20th century with the advent of electronics and later digital technologies, enabling the development of increasingly complex systems.

1.2.1. Definition :

To carry out a system automatic It is perform a or several operations without human intervention.

Example : Machine has wash automatic

Piloting automatic airplane

Note that these systems copy the more often the behavior of the man in the three essential phases from his work:

1st Phase: Observation

2nd Phase: Reflection

3rd Phase: Action

Then back to the essential phase of his work

Example : fill a tank has height data of Water. The three phases are then:

- ✓ Observation of level water current In there tank.
- ✓ Comparison with the level wish.
- ✓ Action on the faucet (opening Or (closing) Then return to an observation phase.

This back constitutes one of the fundamental concepts of In automation, it is also said that a feedback loop was created .

1.2. Terminology of control systems

To understand a closed-loop system, it is essential to master the following terms:

- **Setpoint:** the desired value or target to be reached (e.g., temperature, position).
- **Output** : the actual measured value (current temperature, actual position).
- **Deviation** : difference between the setpoint and the actual output.
- **Command** : action sent to the actuator to correct the deviation.
- **Disturbance** : external element influencing the output (wind, load, etc.).
- **Measurement noise** : error or fluctuation in the measurement of the output.
- **Continuation** : when the instruction changes over time and the system follows.
- **Regulation** : maintaining constant output despite disturbances.
- **Corrector** : a system element that adjusts the control to correct the error.

1.3. Automatic Control Functions

The main functions of automatic control are:

1.3.1Monitoring and security : anomaly detection, emergency shutdown, alerts.

1.3.2 Regulation and control : automatic adjustment of a physical quantity to achieve an objective.

Regulation and control represent two fundamental paradigms in automation concerning the automatic control of physical quantities to achieve specific objectives. These systems belong to the category of closed-loop systems, but differ in their operational purposes.

1.3.2.1 Control Systems:

A closed-loop system is defined as a closed-loop system where the output value is constantly compared to a reference value, generating a deviation signal. This deviation is processed by a controller to generate the command for the operating component.

a) Essential properties:

- ✓ Power amplification between input and output
- ✓ Closed-loop architecture with measurement feedback
- ✓ Transient regime management for dynamic stability
- ✓ Continuous analog signals for input and output quantities

b) Classification of Control Systems

1.3.2.1.1. Follower system control systems

A follower system aims to make the output follow a setpoint that varies over time, the evolution of which can be unpredictable.

Examples:

- trajectory-following robot control
- Radar antenna control
- Autopilot system

1.3.2.1.2 Regulatory systems

A control system aims to maintain an output quantity at a constant value specified by a fixed setpoint, despite external disturbances.

Examples:

- Temperature control in an industrial oven
- Level control in a tank
- Pressure stabilization in a hydraulic circuit

1.4. Concept of system, Loop Open (BO) Loop Closed (BF) :

Automatic can apply has All This Who move, works, se transforms, The application object is called a system.

A system is characterized by its input quantities and of outputs. Input quantities are the quantities that act on the system; there are two types:

Commands : those that one can master
Disruptions : those that one born can not master

A system is said to be open loop (OL) when the control is developed without the aid of knowledge of the output quantities: there is no feedback.

In a closed-loop (CL) system, the control is then a function of the setpoint and of The output, to observe the output quantity we use sensors, it is the information from these sensors that will allow us to develop the control.

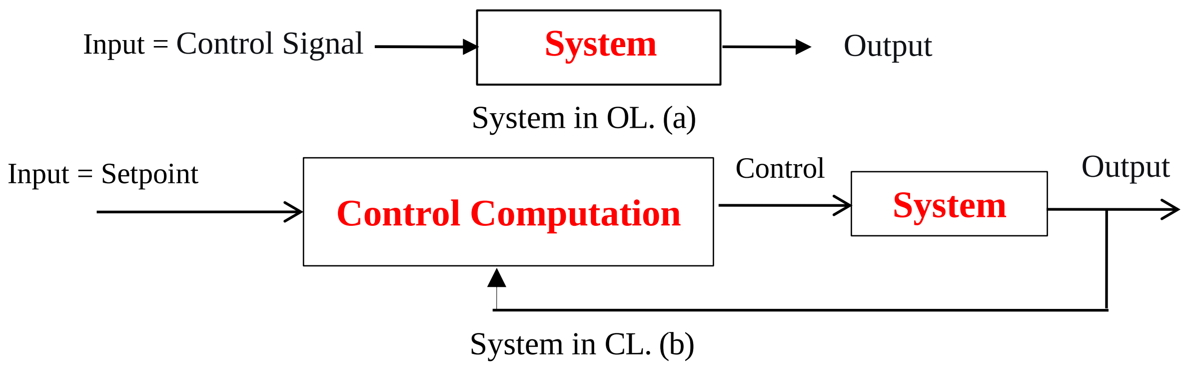


Figure 1.1 : System loop open (OL). (a) **and system in loop closed (CL).** (b)

1.5. Structure and organs of a control system

A servo system East A system loop of which the general structure East data as follows :

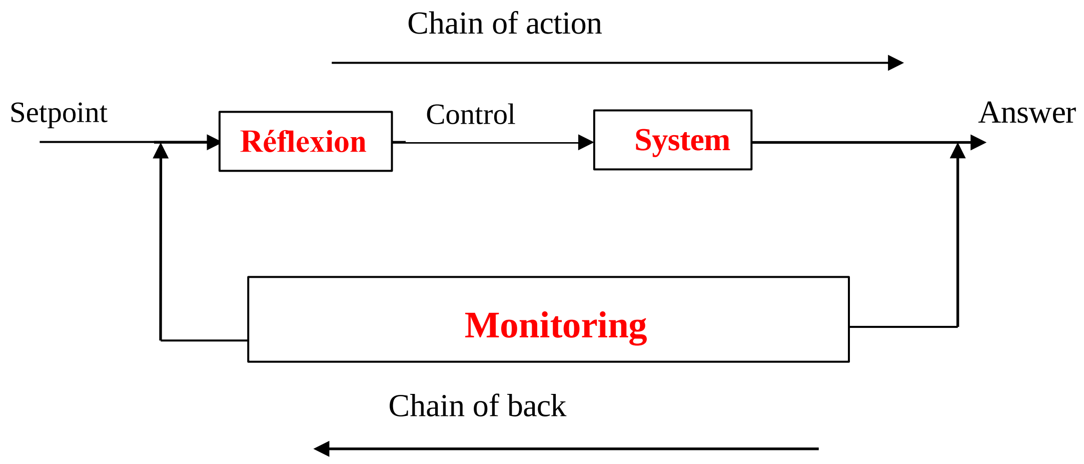


Figure 1.2: Structure of a servo system

Example : of regulation automatic of level water In a tank with leak

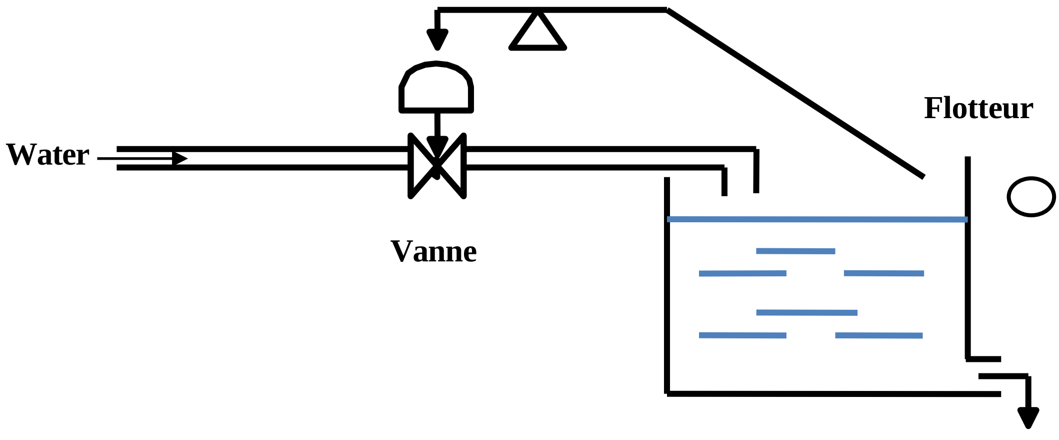


Figure 1.3: Regulation automatic of level water in a tank with leak

The opening Or there closing of there valve East ordered by there relative position of the float The functioning of this regulation can be describe by the plan general below (Figure 1.4)

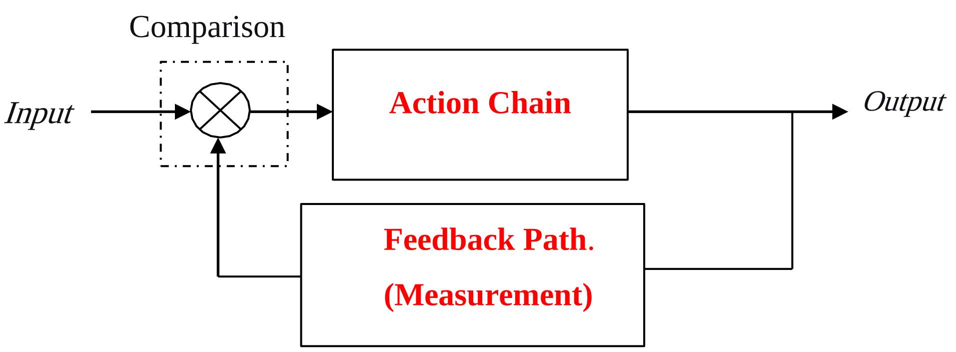


Figure 1.4: Plan general of enslavement

The entrance represents the level water desired, there exit east the level water real, the action se done after comparing the desired level to the actual level.

On represents this enslavement usually by a plan called plan functional or plan block

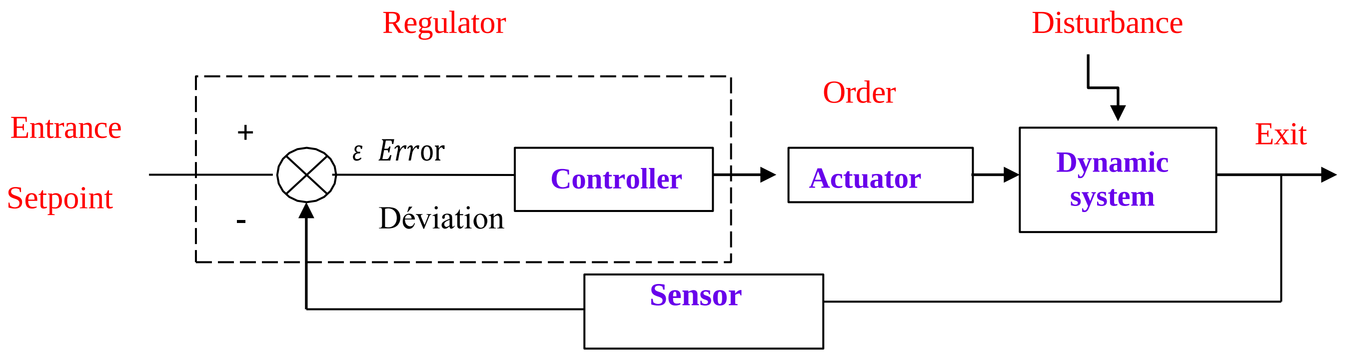


Figure 1.5 : Plan functional of enslavement

a) The regulator (comparator + corrector) generates the control command from the error signal ε : it's the intelligent organ.

b) The actuator or the acting element generally provides the power necessary to perform the task; it is the muscular element (the valve).

c) The dynamic system evolves according to the action, following its own specific physical laws. ; the output is generally a physical quantity that characterizes the task to be performed, in addition this output can fluctuate depending on unpredictable disturbances.

d) The sensor delivers from the output a quantity characterizing the observation (sensor accuracy implies system accuracy).

Example:

The objective is to maintain a constant room temperature by means of a metallic strip, which functions to open or close an electrical contact based on the temperature, thereby supplying or cutting off power to the heating resistor.

Represent this system using a functional block diagram?

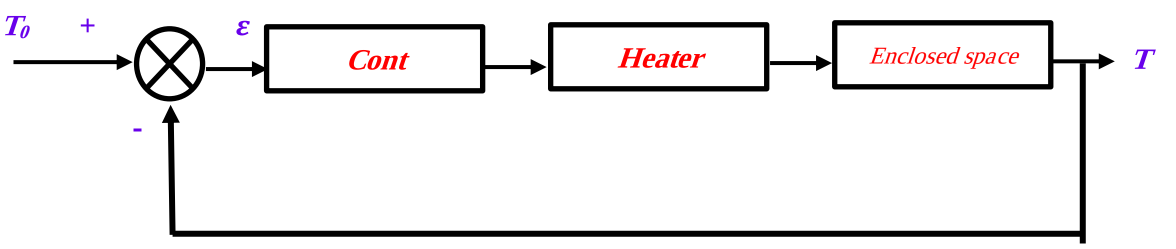


Figure 1.6 : Plan functional of an electrical system

Chapter 2 : Reminders on there transformed of Laplace

2.1 Introduction :

The Laplace transform is an integral operation that transforms a function of a real variable into a function of a complex variable. Through this transformation, a linear differential equation can be represented by an algebraic equation. It also allows for the representation of specific functions (Heaviside distribution, Dirac distribution, etc.) in a very elegant way. It is these possibilities that make the Laplace transform interesting and popular among engineers. This transformation gave rise to the technique of operational calculus or symbolic calculus, which facilitates the resolution of linear differential equations that will represent the systems we will study.

2.2 Definition :

Consider a function real of a variable real *st)* such that *s(t)=0* For *t<0.*

On defines its transformed of Laplace *L(s(t))* as there function ***S*** of there variable complex *P* As follows:

$$S(p) = \int_0^{+\infty} s(t)\, e^{-pt} dt \quad (2.1)$$

2.3 Fundamental Properties of the Laplace Transform:

The following properties allow us to easily calculate (without using the definition of the Laplace transform) the Laplace transforms of certain functions.

a. Linearity :

Let f, g two functions, $\alpha, \beta,$ k constants real

$$(\ \alpha f(t) + \beta g(t)\ ) = \alpha L\ (\ f(t)\ ) + \beta L(g(t)) \quad (2.2)$$

In particular

$$\text{L (f(t)} + g(t)\ ) = L\ (\ f(t)\ ) + L(g(t)) \quad (2.3)$$

$$L\ (\ kf\ \ (t)\ ) = k\ L\ (f(t)) \quad (2.4)$$

b. Laplace Transform of a Derivative:

$$L\left(\frac{df(t)}{dt}\right) = pF(p) - f(0) \quad (2.5)$$

Of even, there Laplace transform for the second derivative (n=2) East :

$$L\left(\frac{d^2f(t)}{dt^2}\right) = p^2F(p) - pf(0) - \frac{df(0)}{dt} \quad (2.6)$$

NB :

- It should be noted that these expressions contain the initial conditions, that is to say the values in *t=0* of the derivatives successive orders inferior has the order of derivation considered
- In the case Or these condition initials are useless, This Who has a priori very often the case The following relationships can be simply retained:

$$\frac{df}{dt} \longrightarrow pF(P) \qquad \frac{d^n f}{dt^n} \longrightarrow p^n F(P) \tag{2.7}$$

c. *Laplace Transform of an Integral:*

$$L(\int f(t)\,dt) = \frac{F(p)}{p} \tag{2.8}$$

d. *Scaling Property:* Either k ∈ ℝ with k ≠ 0

$$\begin{cases} L\big(f(kt)\big) = \frac{1}{k} F\left(\frac{P}{K}\right) \\ L\left(f\left(\frac{t}{K}\right)\right) = k\,F(kp) \end{cases} \tag{2.9}$$

e. *Theorem of delay :*

Consider the function $f(t-\tau)$, in other words, the function $f(t)$ to which we have done undergo a change in the origin of time (figure 2.1), in other words a delay of one time τ.

$$L\big(f(t-\tau)\big) = F(p)e^{-p\tau} \tag{2.10}$$

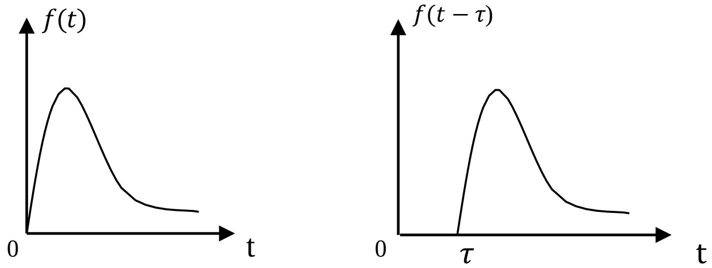


Figure 2.1: Representation temporal of a signal delayed

Demonstration:

$$L\big(f(t)\big) = \int_0^{+\infty} f(t)\, e^{-pt} dt \tag{2.11}$$

Performing In this complete the change of variable $u = t + \tau$

$$L\big(f(t-\tau)\big) = \int_\tau^{+\infty} f(u-\tau)\, e^{-p((u-\tau))} du \tag{2.12}$$

In noticing that there function $f(u-\tau) = 0 \; pour \; t < \tau$, on can, without to change there value of the integral, choose a lower bound of integration lower than τ.

$$F(P) = \int_0^{+\infty} f(u-\tau)\, e^{-pu)} e^{p\tau} du \tag{2.13}$$

$$F(P) = e^{p\tau} \int_0^{+\infty} f(u-\tau)\, e^{-pu} du \qquad (2.14)$$

By definition, $\int_0^{+\infty} f(u-\tau)\, e^{-pu} du$ it is the transform of Laplace of $f(u-\tau)$

Hence: $$L\big(f(t-\tau)\big) = e^{-p\tau} F(P) \qquad (2.15)$$

***f. Initial Value Theorem*:**

$$\lim_{n\to 0^+} f(t) = f(0^+) = \lim_{p\to +\infty} pF(P) \qquad (2.16)$$

***g. Final Value Theorem*:**

$$\lim_{t\to \infty} f(t) = \lim_{p\to 0} pF(P) \qquad (2.17)$$

2.4 Inverse Laplace Transform:

Just as a function of time can have a Laplace transform, it is possible has leave of a function $F(P)$ of find her original, otherwise said there inverse Laplace transform:

$$f(t) = \int_{-\infty}^{+\infty} F(P)\, e^{pt} dt \qquad (2.18)$$

Transformed of Laplace of a few signals usual :

2.4.1 Unit Step:

The level unit (Figure 2.2) is the function $u(t)$ such that :

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

On has so: $$U(P) = L(u(t)) = \frac{1}{p} \qquad (2.19)$$

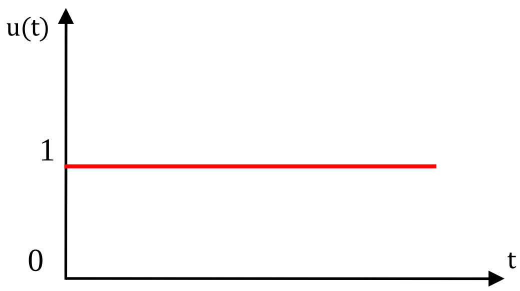


***Figure* 2.2: Echelon unit**

Account tenuous of there linearity of there transformed of Laplace, All level (non -unit) of amplitude A, will have the Laplace transform

$$F(P) = L(f(t)) = L(Au(t)) = \frac{A}{p} \qquad (2.20)$$

2.4.2 Ramp or Velocity Step:

He it is in reality of the complete of there function $u(t)$previous, on there note generally $v(t)$, such that $v(t)= t\ u(t)$(figure 2.3).

$$v(t) = \begin{cases} 1 & t > 0 \\ 0 & t \leq 0 \end{cases}$$

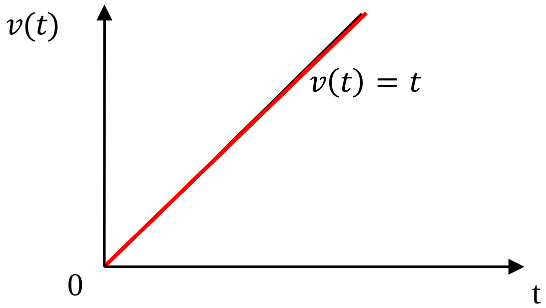


Figure 2.3 Ramp

On has obviously $$V(P) = L(v(t)) = \frac{1}{p^2} \quad (2.21)$$

2.4.3 Unit Impulse:

In drifting there function $\boldsymbol{u(t)}$, on obtains a function usually noted $\boldsymbol{\delta(t)}$ which is called unitary impulse or Dirac impulse

$$\delta(t) = \frac{du(t)}{dt} \quad (2.22)$$

This is a zero function everywhere except for t=0 where it has an infinite value.

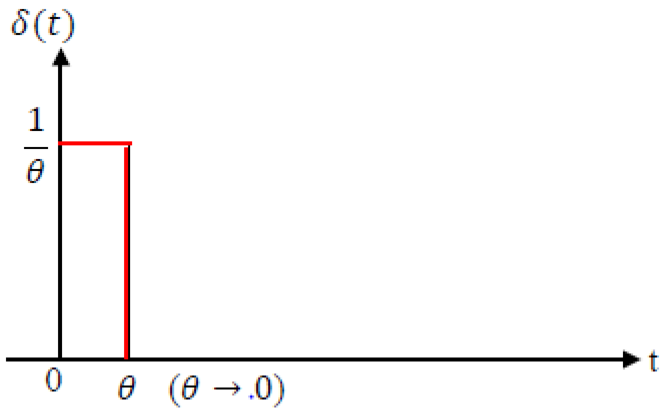


Figure 2.4 : Model of the impulse of Dirac

$$\Gamma(p) = L(\delta(t)) = 1 \quad (2.23)$$

2.4.4 *Sinusoidal Signal*

On considered A signal $s(t) = \sin(\omega t + \varphi)$ For $t \geq 0$.

$$S(p) = L(s(t)) = \frac{p sin\varphi + \omega cos\varphi}{p^2 + \omega^2}$$

On will retain basically the two results following:

For $s(t) = \sin\omega t \longrightarrow S(p) = \frac{\omega}{p^2+\omega^2}$ (2.24)

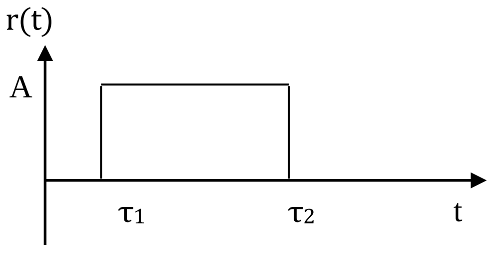
And for $s(t) = \cos\omega t \longrightarrow S(p) = \frac{p}{p^2+\omega^2}$ (2.25)

2.4.5 Arbitrary Signals

Given any signal, one can certainly undertake the direct calculation of the transform of Laplace. This calculation can Sometimes be relatively delicate. On can Also refer to a Laplace transform table such as the one provided in the appendix.

Example 1:

Either r(t) given graphically

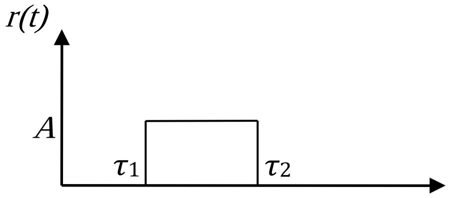


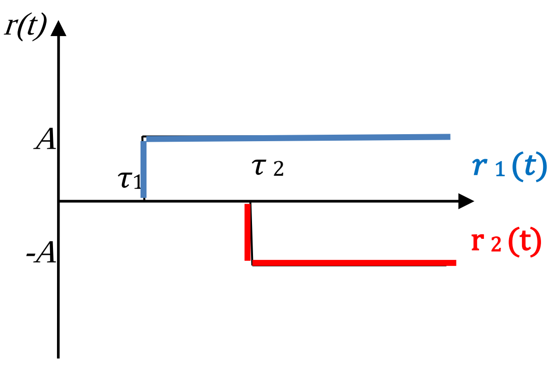
1) Calculate $L[r(t)]$.

2) What is the expression of$L[r(t)]$ In the case particular $\tau_1 = 0\,, \tau_2 = \tau$.

3) Watch that if $A = \frac{1}{\tau}$ And if $\tau \longrightarrow 0$ SO $L\big(r(t)\big)= L\big(\delta(t)\big)= 1$

Solution :





1) $L[r(t)] = L[r_1(t)] + L[r_2(t)]=\frac{A}{P}e^{-\tau_1 p} - \frac{A}{P}e^{-\tau_2 p}$ (2.26)

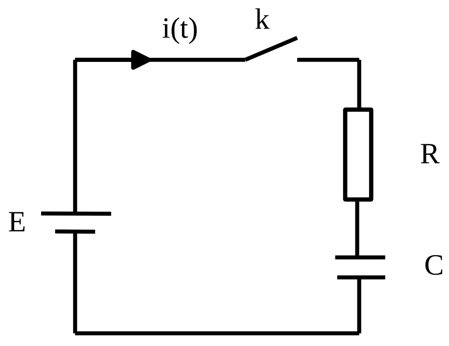
2) *If* $r_1 = 0 \;\; et \;\; r_2 = \tau$ then $R(p) = A\,\frac{1-e^{-\tau p}}{p}$ (2.27)

3) *If* $A = \frac{1}{\tau}$ then $R(p) = A\,\frac{1-e^{-\tau p}}{\tau p}$ (2.28)

When $\tau \longrightarrow 0, r(t) \longrightarrow \delta(t)$

$\tau \longrightarrow 0\ ;\ e^{-\tau p} \approx 1 - \tau p \Rightarrow R(p) = \frac{1-(1-\tau p)}{\tau p} = \frac{\tau p}{\tau p} = 1$, donc $L(\delta(t)) = 1$.

Example 2 : Either the circuit following :



E = constant and in the initial state, C is charged to q.

1) Write the equations governing the circuit.

2) Calculate the initial and final values of i(t).

3)Comment on your results

2.5 Modeling of Linear Control Systems

2.5.1 Representation by differential equations:

Dynamic systems, whether hydraulic, mechanical, or electrical, can be described by differential equations which are established based on the laws of physics that govern the system under consideration, for example Kirchhoff's law for electrical systems, Newton's law for mechanical systems. The mathematical description of a given system is called a mathematical model.

2.5.1.1 Examples of modeling

a-1 Electrical system (first order)

Consider the RC circuit in Figure 2.5. We consider it to be a system with an input e(t) and a exit s (t) , on can apply there second law of Kirchhoff thus that there law Faraday 's law .

$$e(t) = Ri(t) + s(t) \tag{2.29}$$

$$i(t) = C\frac{ds(t)}{dt} \tag{2.30}$$

Substitute (2.30) in (2.29) on obtains the equation differential

$$e(t) = RC\frac{ds(t)}{dt} + s(t) \tag{2.31}$$

This system is therefore modeled by a first-order differential equation. It is also called that the system is first-order.

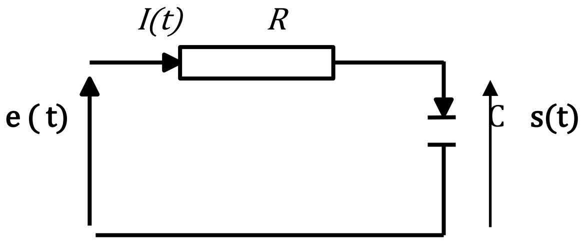


Figure 2.5: Circuit RC

a-2 System electric (second order)

On considered now the system presenting a circuit RLC as the watch (the Figure 2.6). We can write using the same principle as for the previous example

$$e(t) + L\frac{di(t)}{dt} + Ri(t) + s(t) = e(t) \quad (2.32)$$

$$i(t) = C\frac{ds(t)}{dt} \quad (2.33)$$

As previously, on can get a equation differential connecting there exit s(t) at the entrance e(t) :

$$LC\frac{d^2s(t)}{dt^2} + RC\frac{ds(t)}{dt} + s(t) = e(t) \quad (2.34)$$

The equation Thus found East a equation differential of second order Who model a second-order electrical system.

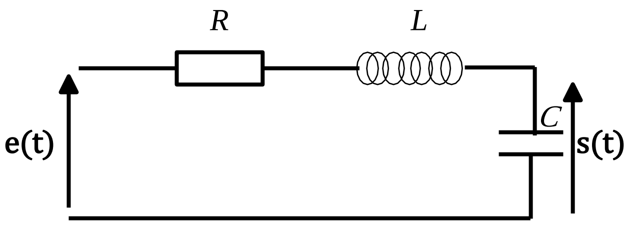


Figure 2.6 Circuit RLC

b. ***Mechanical Second-Order System***

On considered the system mechanical of there figure 2.7 , a mass is subject has a force *e (t)* , at a force of reminder of spring $\vec{f}_k = -k\vec{x}(t)$ And a strength of friction viscous $\vec{f}_h = -f\frac{d\vec{x}}{dt}$

The relationship fundamental of there dynamic we given:

$$M\frac{d^2x(t)}{dt^2} = e(t) - f\frac{dx(t)}{dt} - kx(t) \quad (2.35)$$

Alternatively,

$$M\frac{d^2x(t)}{dt^2} + f\frac{dx(t)}{dt} + kx(t) = e(t) \quad (2.36)$$

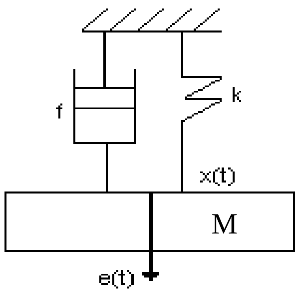


Figure 2.7: Example of a system of 2nd mechanical order

c. *Electromechanical System*

This is a model of a system for controlling the speed of a DC motor. This model can represent that of a motor controlling a joint on a robot arm. It is very common in enslavement. Its diagram is as follows:

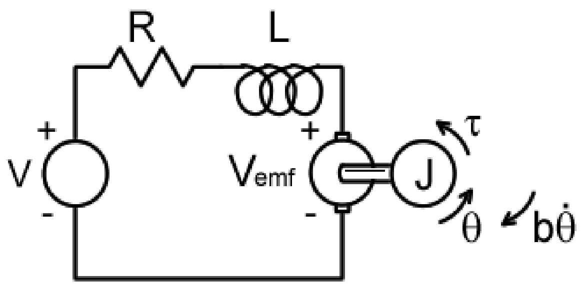


Figure 2.8 : Model of an engine has fluent continuous

The motor is powered by a voltage *V*. The motor with its winding is equivalent to an electrical circuit with resistance *R*, inductance L, and back electromotive force *Vemf* . The motor drives a mechanical system with moment of inertia *J* , including the rotor and the load. Presence of a friction coefficient *b* . the system mechanical turned of a corner θ.
We consider that the constants are such that ***K t*** (constant characterizing the reinforcement) is equal to ***K e*** (motor constant): $K=K_e=K_t$.

- **Determine the entrance and the exit.**
  The output quantity is the angular position of the joint, the angleθ that we wish to control. The input quantity of the system is the voltage V applied to the motor.
- **Find the link between there exit and the entrance** .

The torque applied by the motor, ***T*** , is proportional to the armature current, **i** , by a constant factor **Kt .** The back electromotive force **,** ***Vemt*** , is related to the rotational speed by the following equations:

$$T = k_t . i(t) \tag{2.37}$$

$$e= k_e . \frac{d\theta}{dt} \tag{2.38}$$

Applying Newton's and Kirchhoff's laws, we obtain the following equations:

$$J\ddot{\theta} + b\dot{\theta} = ki(t) \tag{2.39}$$

$$L\frac{di(t)}{dt} + Ri(t) = V - K\dot{\theta} \quad (2.40)$$

2.6 Representation of Control Systems Using Transfer Functions

- ***Definition***

Consider A system linear any possessing a input *e(t)* And a exit *st)* .

Figure 2.9 : System

On assumed that he East governed by a equation differential of degree *n*

$$a_n\frac{d^n s(t)}{dt^n} + \dots_{\dots} a_1\frac{ds(t)}{dt} + a_0 s(t) = b_m\frac{d^m e(t)}{dt^m} + \dots_{\dots} b_1\frac{de(t)}{dt} + b_0 e(t) \quad (2.41)$$

With $\mathrm{n}, \mathrm{m} \in \mathbb{N}, \mathrm{m} \leq \mathrm{n}$

The coefficients $a_n, a_{n-1}, \dots\dots, a_1, a_0, b_m, b_{m-1}, \dots b_1, b_0$ are constants (linear system).
If We let's apply there transformation of Laplace to two members of this equation,
All assuming the various initial conditions are zero.

$$a_n P^n S(p) + \dots\ a_1 pS(P) + a_0 S(P) = b_m p^m E(P) + \dots\ b_1 p E(\ ) + b_0 E(p) \quad (2.42)$$

Either

$$(a_n P^n + a_{n-1}p^{n-1} + \cdots + a_1 p + a_0)S(P) = (b_m P^m + a_{m-1}p^{m-1} + \cdots + b_1 p + b_0)E(P)$$

Hence :

$$\frac{S(P)}{E(P)} = \frac{(b_m P^m + a_{m-1}p^{m-1} + \cdots + b_1 p + b_0)}{(a_n P^n + a_{n-1}p^{n-1} + \cdots + a_1 p + a_0)} \quad (2.43)$$

This fraction rational of two polynomials of there variable complex p East called Transfer Function (TF) or system transmittance, commonly denoted:

$$G(p) = \frac{S(p)}{E(p)} \quad (2.44)$$

As this function East a function rational of two polynomials in *p* , he East possible to factor these two polynomials in the field of complex numbers. We obtain:

$$G(p) = \frac{b_m(p - z_m)(p - z_{m-1})\dots(p - z_0)}{a_n(p - p_n)(p - p_{n-1})\dots(p - p_0)} = \frac{b_m\prod_{i=1}^{m}(p - z_i)}{a_n\prod_{j=1}^{n}(p - p_j)}$$

(2.45)

The roots z_i numbers that make the numerator zero are called zeros. of the transfer function. The roots p_i, which cancel out its denominator these, are the poles of the transfer function. These parameters can be complex or real. The degree *n* of the polynomial in the denominator is called the order of the system.

Noticed :

He east often preferable of master in evidence the gain K of system Thus that the number α of integrators pure also called kind of system.

$$T(p) = k\frac{1}{p^{\alpha}}\,\frac{1+\cdots+c_m p^m}{1+\cdots d_{n-\alpha}p^{n-\alpha}} \tag{2.46}$$

This last shape can Sometimes se find below shape factored (shape canonical) or Bode form:

$$T(p) = k\frac{1}{p^{\alpha}}\,\frac{(1+\tau_1' p)\ldots(1+\tau_m' p)}{(1+\tau_1 p)\ldots(1+\tau_{n-\alpha} p)} \tag{2.47}$$

In this formulation, the τ And τ' are similar has of the constants of time.

- ***Example function of transfer (system electric)***

We let's start again the example of paragraph 6.1.1 We airplanes seen that :

$$e(t) = RC\frac{ds(t)}{dt} + s(t) \tag{2.48}$$

Applying the transformation of Laplace to two members of this equation, all in assuming the various initial conditions are zero

$$RCpS(p) + S(p) = E(p) \tag{2.49}$$

$$(RCp+1)S(p) = E(p) \Rightarrow \frac{S(p)}{E(p)} = \frac{1}{1+RCp} \tag{2.50}$$

We can form the function of transfer of this system: $G(p) = \frac{S(p)}{E(p)} = \frac{1}{1+RCp}$

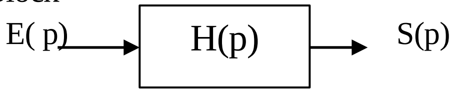
2.7 Functional Block Diagrams:

2.7.1 Definition :

A functional diagram is a simplified representation of a process's transfer function. In other words, it's a graphic that can incorporate elementary symbols such as summing junctions, comparators, sensors, etc.

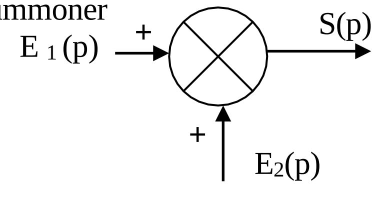
There is four elementary diagrams used in the Functional representation of control systems:

- Block



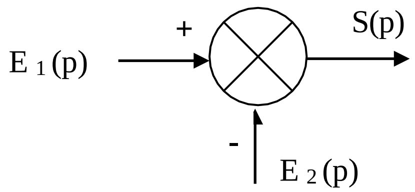
S(p)=H(p)E(p)

- Summoner



S(p)=E 1 (p)+ E 2 (p)

- Comparator



$S(p) = E_1(p) - E_2(p)$

2.7.2 Example of a Functional Block Diagram:

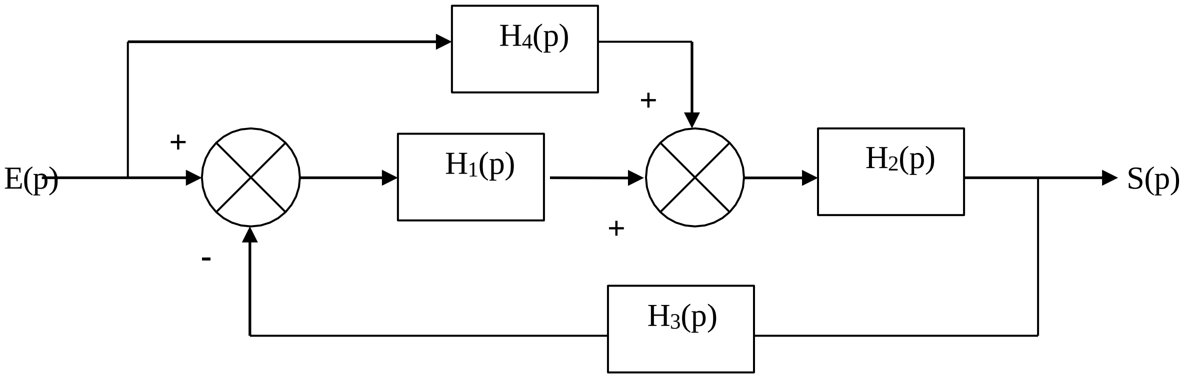


Figure 2.10 : Example of a Block Diagram

Noticed :

- Representation of a Function of Transfer in Loop Open (FTBO) :

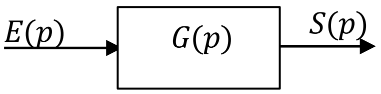


Figure 2.11 : Block Diagram of an FTBO

.
$$S(p) = G(p)E(p) \Leftrightarrow FTBO = G(p) = \frac{s(p)}{E(p)} \tag{2.51}$$

- Representation of a Function of Transfer in Loop Closed (FTBF)

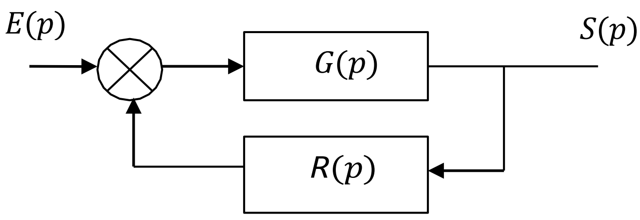


Figure 2.12: Block Diagram of an FTBF

$$S(p) = G(p)\mathcal{E}(p) \tag{2.52}$$

$$\mathcal{E}(p) = E(p) - R(p)S(p) \tag{2.53}$$

Substitute the equation (2.53) In (2.52) on find:

$$S(p) = G(p)(E(p) - R(p)S(p)) \qquad (2.54)$$

$$FTBF = \frac{S(p)}{E(p)} = \frac{G(P)}{1+G(p)R(p)} \qquad (2.55)$$

Hence, for a system has back unitary $R(p) = 1$ there FTBF becomes:

$$FTBF = \frac{S(p)}{E(p)} = \frac{G(P)}{1+G(p)} \qquad (2.56)$$

2.7.3 Reduction of a Functional Block Diagram:

Simplifying or reducing a diagram involves transforming it to highlight the transfer function. To do this, we will use a number of basic rules, which are:

2.7.3.1 Rule 1 :Blocks in Series (Cascade)

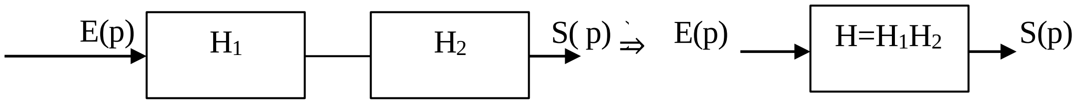


Figure 2.13: Blocks in cascade

2.7.3.2 Rule 2 : Blocks in parallel :

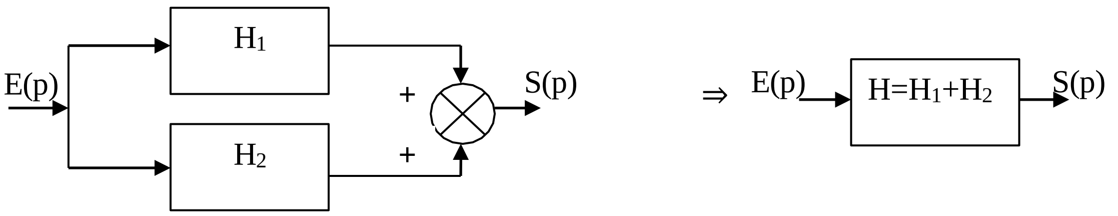


Figure 2.14: Blocks in parallel

2.7.3.3 Ruler 3 :***Black's Formula (Loop Reduction):***

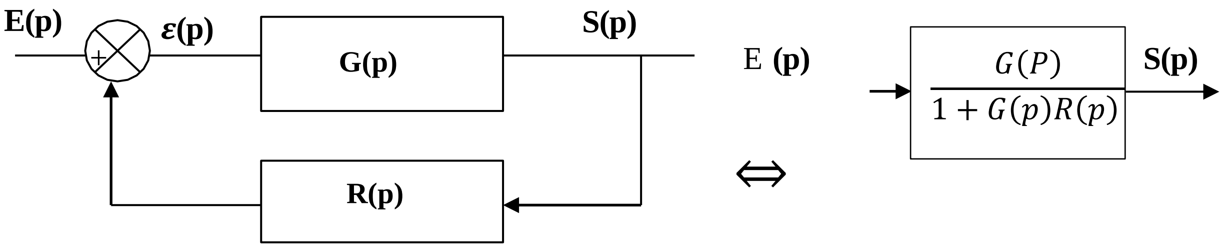


Figure 2.15: Representation of a closed loop

2.7.3.4. Ruler 4 : Transformation of a Comparator into a Summing Point :

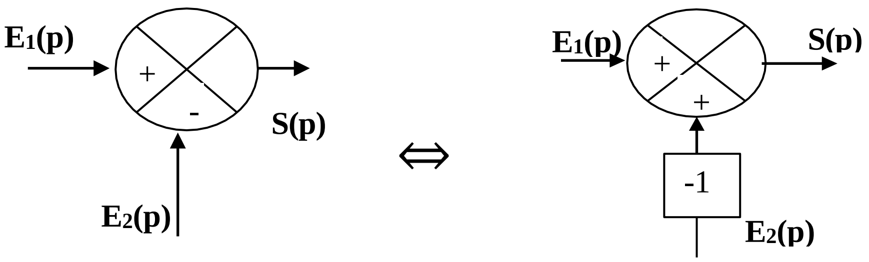


Figure 2.16: Transformation of a comparator in a summer

2.7.3.5 Ruler 5 : Relocation of a Comparator:

a) Moving a Downstream Block Upstream:

This type of displacement results in the addition of a functional block with an equal transfer function has the entrance of summer who takes there place of comparator as the watch there Figure 2.17:

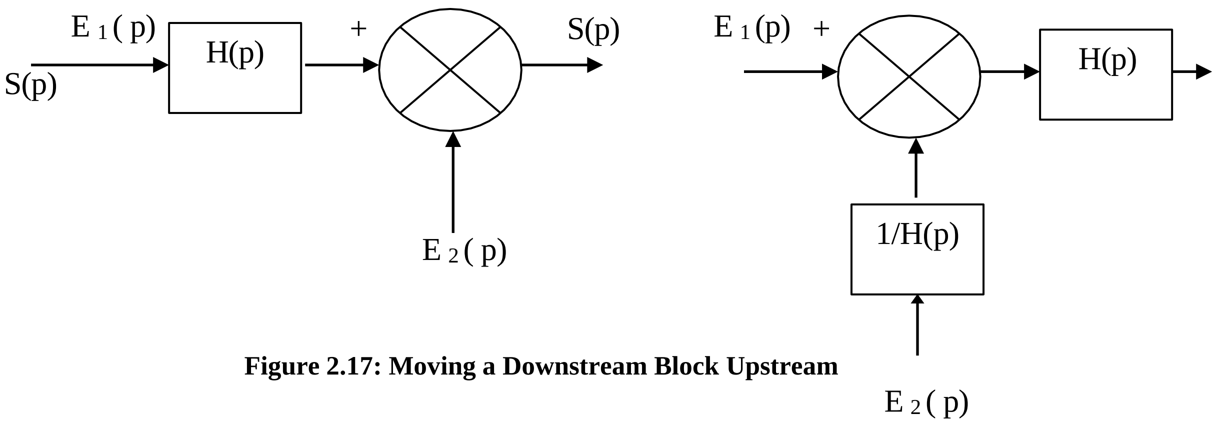


Figure 2.17: Moving a Downstream Block Upstream

$$S(p)=E_1(p)H(p)+E_2(p)=H(p)[E_1(p)+(1/H(p))E_2(p)] \quad (2.57)$$

b) Moving an Upstream Block Downstream

By a reasoning analogous to case previous on obtains:

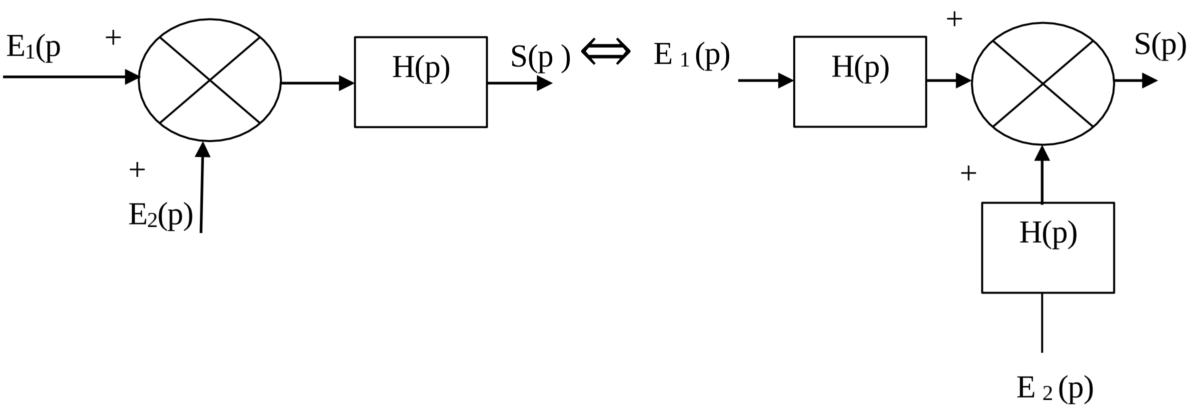


Figure 2.18: Moving an Upstream Block Downstream

$$S(p)=[E_1(p)+E_2(p)]H(p)=H(p)E_1(p)+H(p)E_2(p) \quad (2.58)$$

2.3.7.6 Rule 6 : Relocation of a Sensor:

Shift of a block downstream in upstream

:

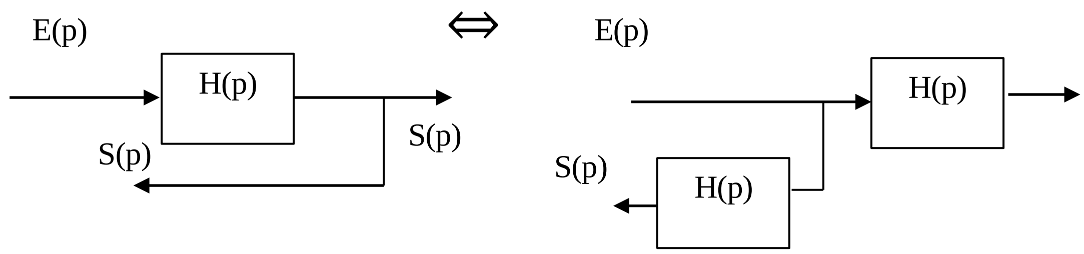


Figure 2.19: Shift of a block downstream in upstream

Shift of a block upstream in downstream:

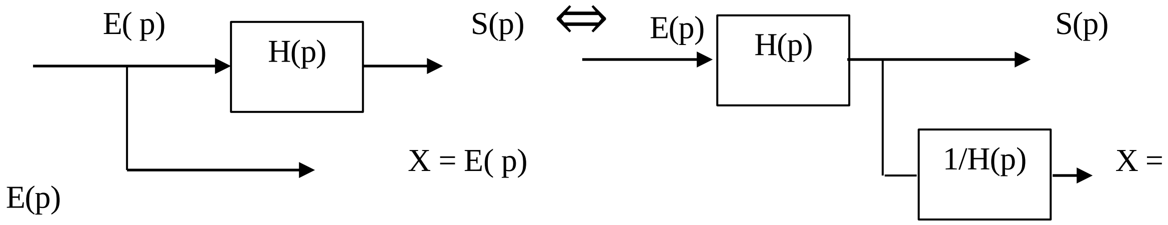


Figure 2.20: Shift of a block upstream in downstream

2.7.3.7 Rule 7 : *Permutation of the sensors :*

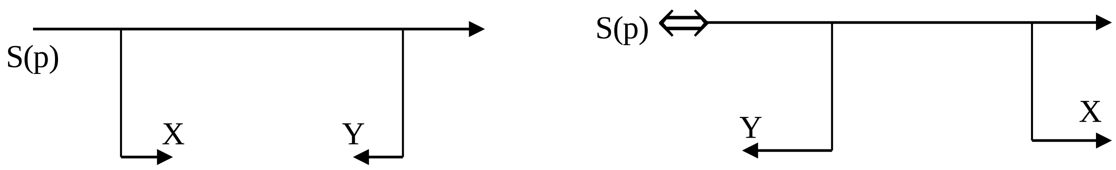


Figure 2.21: Permutation of the sensors

2.7.3.8 Rule 8 : *Permutation of the summarizers* :

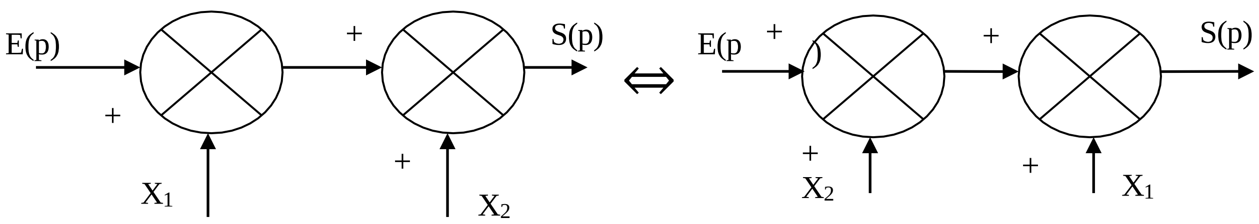


Figure 2.22: Permutation of summers

Example :

Either the plan block following

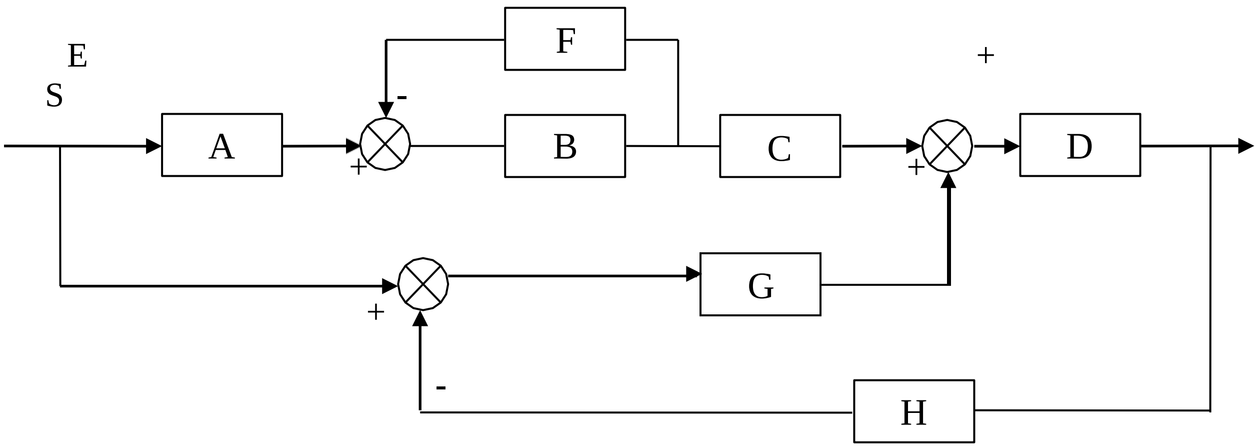
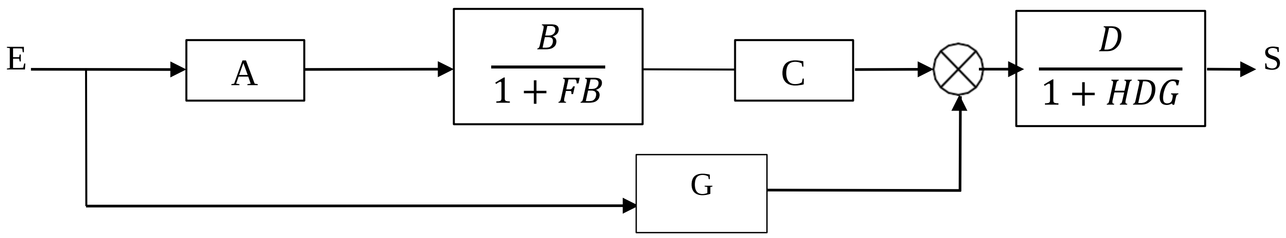
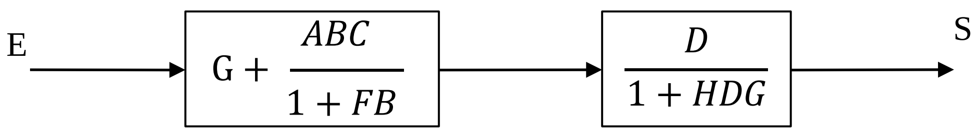


Figure 2.22:Example of a functional diagram

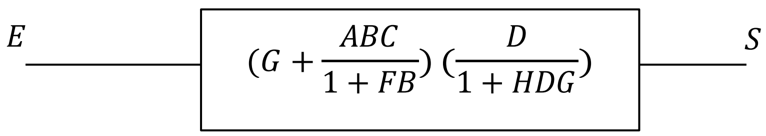
Calculate its function of transfer S/E?

Solution:





In end: there putting cascading



So there function of transfer:

$$\frac{S}{E} = \frac{DG + FBDG + ABCD}{(1 + FB)(1 + HDG)}$$

Chapter 3: Performance Characteristics of a Feedback Control System

3.1. Definitions:

Analyzing the functioning of a dynamic system allows us to distinguish two distinct phases:
3.1.1 ***Steady state:*** the output value should be as close as possible to the desired value. In reality, a slight error always remains.

3.1.2 ***Transient regime:*** the system evolving between two permanent regimes, the time taken by the system to go from one to the other and the way in which it reaches the final state, are very important.

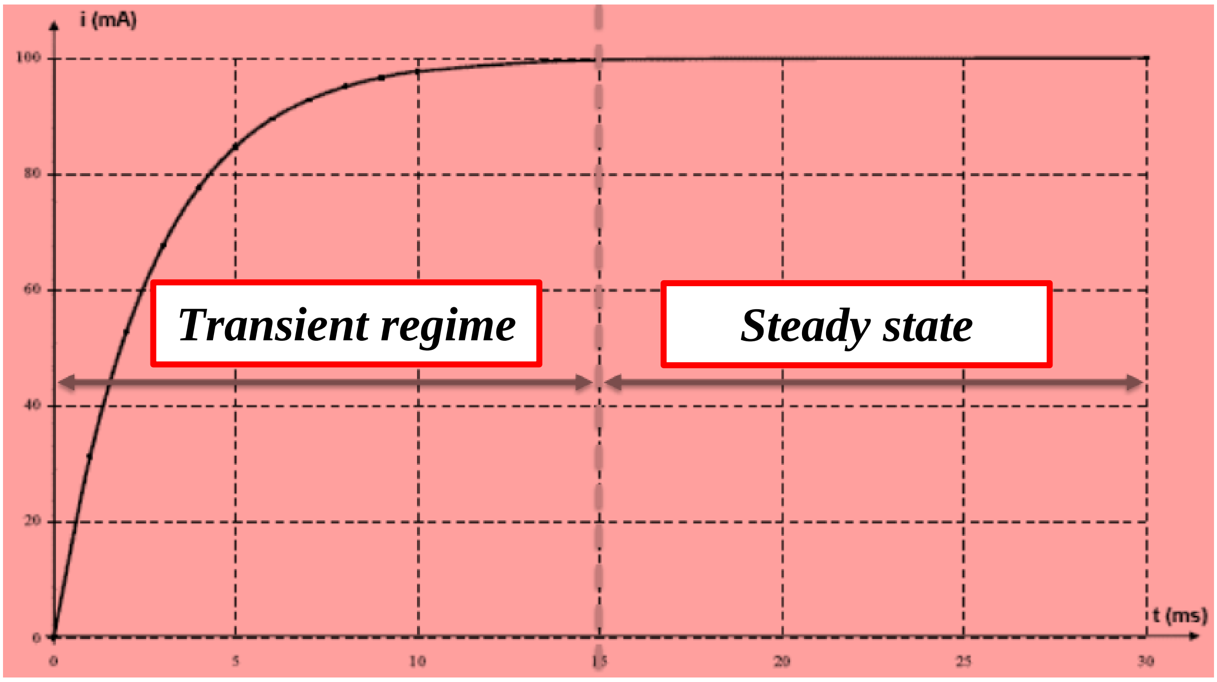


Figure 3.1: Representation of the steady state and transient regimes of an electrical system

3.1.3 ***Concept of stability:*** A linear system is said to be stable if, after subjecting its input to a sudden change (unit step, for example):

- ✓ static error or permanent deviation when the input quantity is a constant; for an ideal system, it must be zero.
- ✓ dragging error when the input quantity is a linear function of time

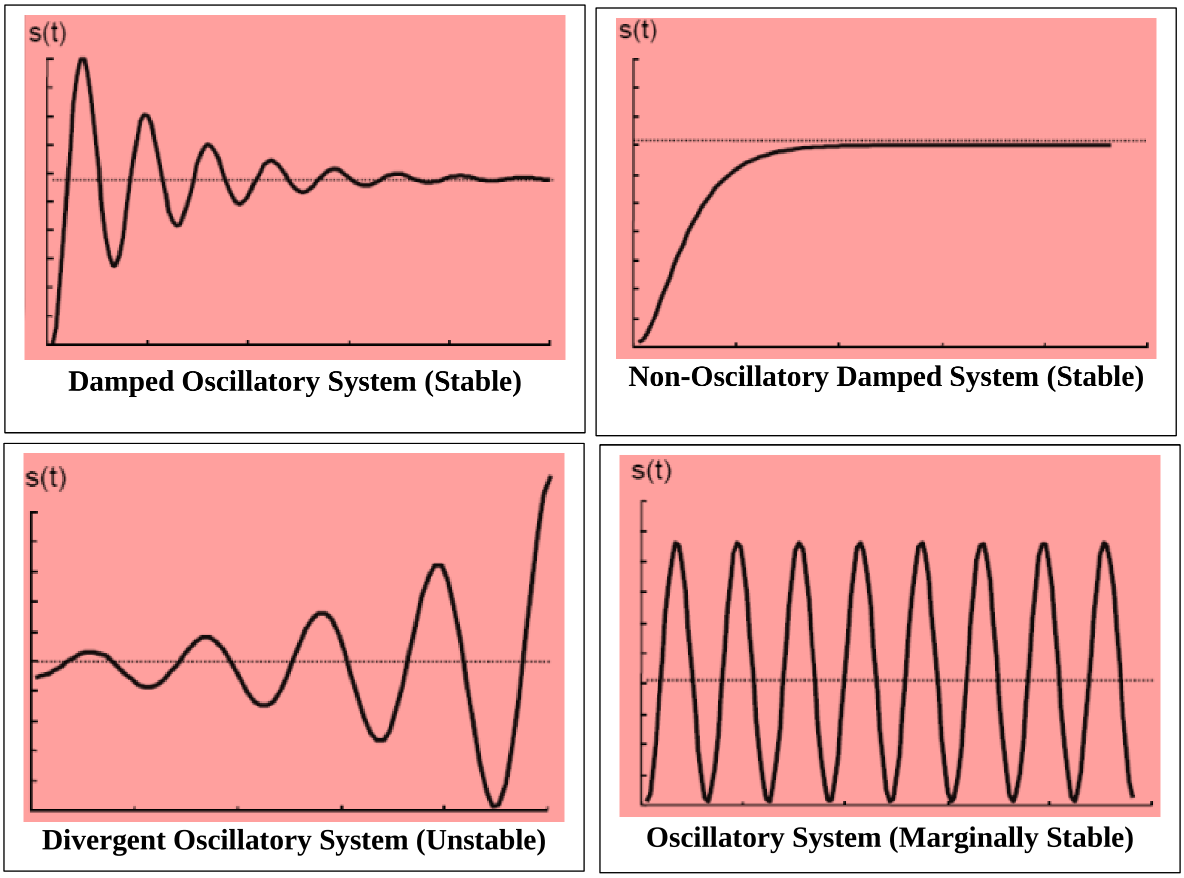


Figure 3.2: The different stability regimes

3.1.4 ***Speed:*** The system reaches its final value in the fastest possible time. This practically reflects the transient duration. More precisely, it is expressed by the response time Te or settlement time, which is the time taken for the measurement to reach its final value within ±5% of its variation while remaining within this ±5% range.

Speed = response time

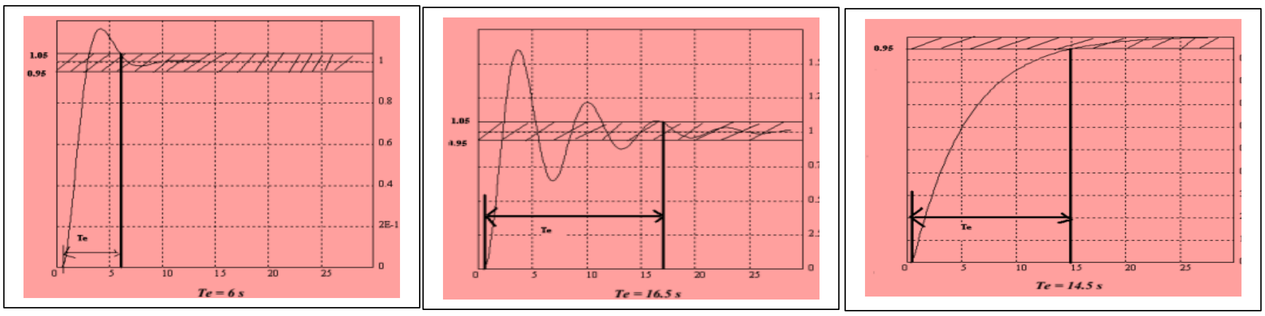
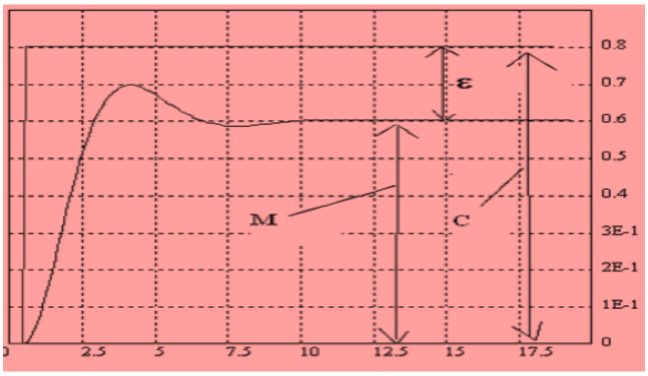


Figure 3.3 : The three types of temporal response

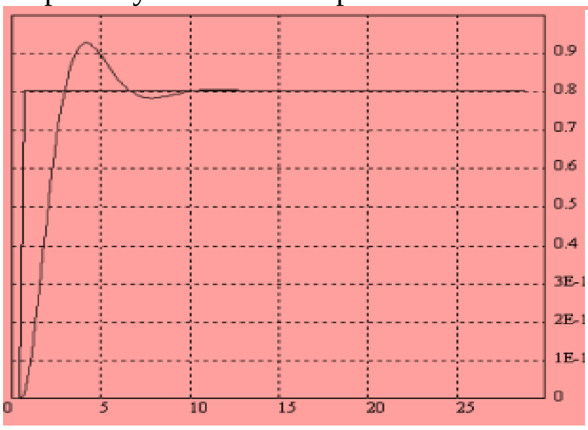
3.1.5 ***Accuracy:*** The ability of the control system to reach the setpoint. It is defined based on the εsteady-state error, as shown in the following figure:

Accuracy error (%) = (ε/C).100

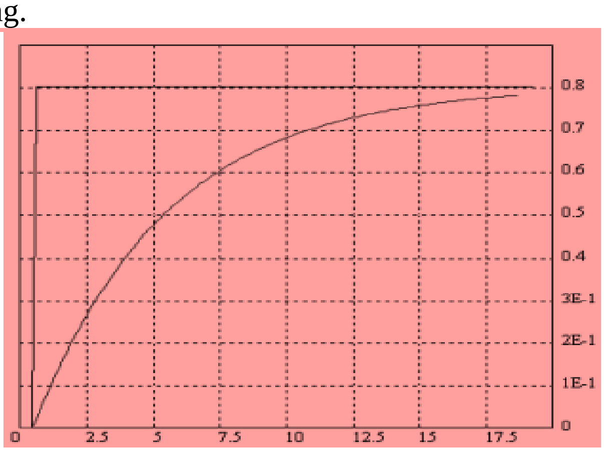
Exemple : pour C = 100 % et ε = 20 % Donc, l'erreur de précision est : (20/100).100 = 20 %



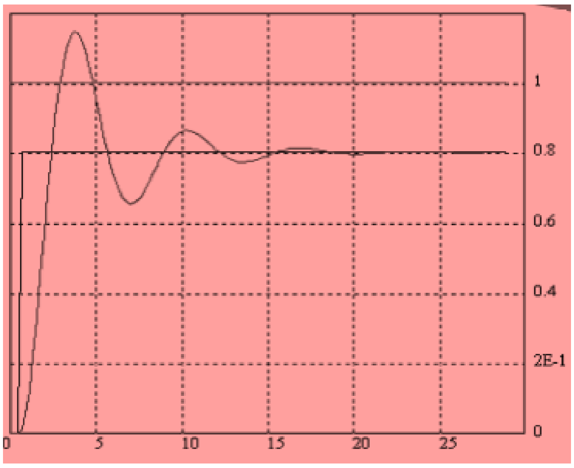
3.1.6 ***Overshoot:*** Often expressed as a percentage (%). Even when a system is stable, the output may exceed the setpoint before stabilizing.



Well-damped system



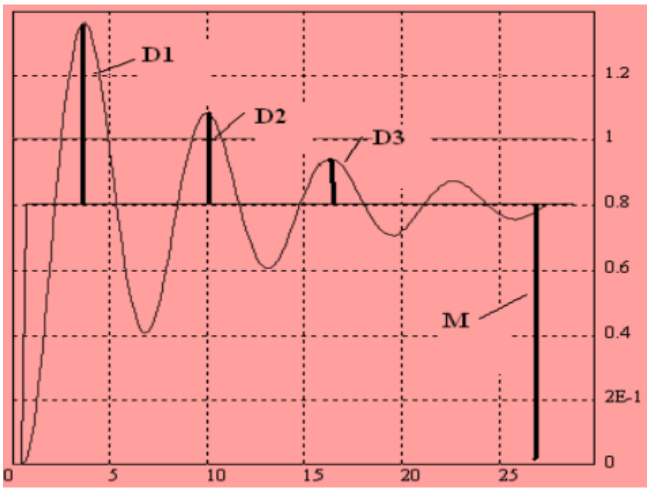
Very well-damped system



System with little cushioning

Figure 3.4: Different Types of Systems

Example: Consider the system described by the following time response:



Calculate the depreciation of this system:
Given that depreciation is generally expressed in two ways:
Depreciation per period = D2 / D1
Exceedance (%) = D1.100 / ΔM
D1, D2 and ΔM are expressed using the same units (mm, %, physical unit)
From the system above, we can derive:
For D1 = (1.36-0.8) = 0.56 V
D2 = (1.09 - 0.8) = 0.29 V
ΔM = 0.8 V

The depreciation per period is: 0.29/0.56 = 0.518.
The excess is: (0.56/0.8) * 100 = 70

3.2 Time Response of Linear Systems:

We want to characterize systems by their transfer function and their behavior. The latter can be demonstrated by their response to a given input. Traditionally, we can learn a great deal about systems by observing their response to the following inputs:

- The impulse : answer impulsive.
- The level : index response
- There ramp : answer has a ramp
- There sinusoid : answer frequency

In the first part, we will establish the link between transfer functions and time responses (that is, impulse responses, step and ramp). In the remainder of the course, we will study the simple and widespread systems of first and second order. Furthermore, the methods used to study these systems can be easily generalized to others.

3.2.1 *Response of first-Order Systems*

3.2.1.1 Transfer Function

A system of first order East describe by the equation differential has coefficients constants

$a_0, a_1, b_0, b_1 \in \mathbb{R}$ such that :

$$b_0 s(t) + b_1 \frac{ds(t)}{dt} = a_0 e(t) + a_1 \frac{de(t)}{dt} \quad (3.1)$$

We do not we will deal with, in this chapter, that the systems for which $a_0 \neq 0$ and $a_1 = 0$

$$T(p) = \frac{a_0}{b_0 + b_1 p} \quad (3.2)$$

That we put below there shape

$$T(p) = \frac{k}{1 + \tau p} \quad (3.3)$$

On call K the static gain and τ there constant of time of the system.

3.2.1.2 Responses to typical signals

In the following section, we will study the responses of a first-order system (with *k=1)* to different input signals.

- ***Step Response***

For all the answers index (has A level) on defines:

- Diet permanent $(t) = (t) \quad \forall t \gg t_r \quad (s_p(t) = lim_{t \to \infty} s(t))$ (3.4)
- Time of ascent t_m East THE time during which S (t) pass of $0.1 s_p(t)$ has $0.9\, s_p(t)$
- Time of answer has $5\% t_r$ is the time at end of which :

$$\forall t > t_r \; (t) - (t) < 0.05\,(t)$$

On applied has the entrance of this system A level amplitude E_0.

$$e(t) = E_0 u(t) \overset{TL}{\rightarrow} E(p) = \frac{E_0}{p} \quad (3.5)$$

There exit of system is such that :

$$S(p) = E(p)\,T(p) = \frac{KE_0}{p(1 + \tau p)} \quad (3.6)$$

And by application of TL $^{-1}$ therefore

$$s(t) = KE_0(1 - e^{-t/\tau}) \quad (3.7)$$

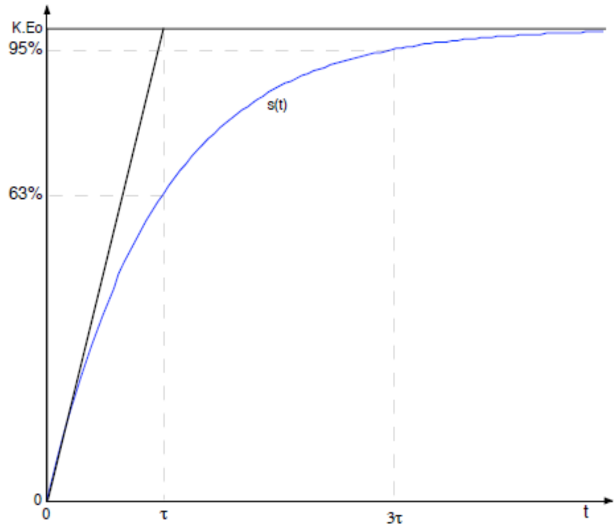


Figure 3.5: ***Step Response of a First-Order System***

On the plotted above (Figure 3.4), on can note

- $s(\tau) = 0.63$ KE $_0$
- $lim_{t\to\infty} s$ (p) $=$ KE $_0$
- There tangent to the origin has a slope of $\frac{kE_0}{\tau}$
- Time of ascent $\approx 2\tau$
- Time of answer has 5% $\approx 3\tau$

On can trace there curve in contact details reduced, that's to say the trace of y $= s$ (t) in function E_0 Of $x = \frac{t}{\tau}$ whom born depends more of τ neither of k neither of l amplitude d ′entrance.

$$(y) = 1 - e^{-x}) \quad (3.8)$$

➢ ***Ramp Response***

The entrance is a ramp : e (t) $=$ a(t) its transformed Laplace East :
$E(P) = \frac{a}{p^2}$, The exit is given by:

$$S(p)\frac{ka}{\tau}.\frac{1}{p^2\left(p+\frac{1}{\tau}\right)} \quad (3.9)$$

$$s(t) = K.a.(t-\tau) + K.a.\tau.e^{-\frac{t}{\tau}} \quad (3.10)$$

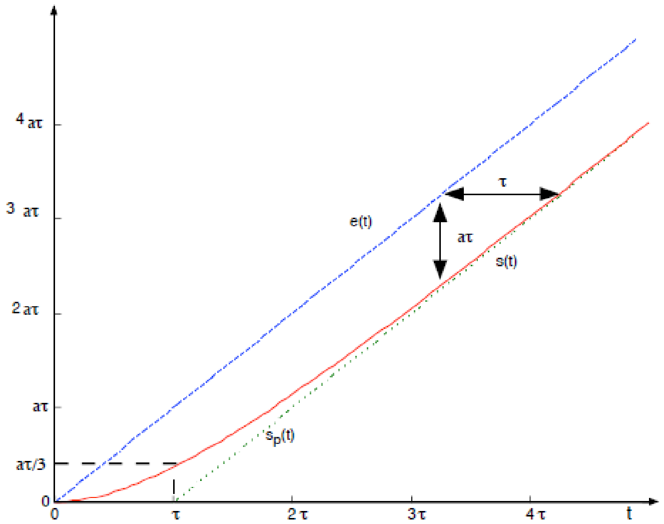


Figure 3.6: Ramp Response of a First-Order System

The characteristics of this answer are :

- The diet permanent East e (t) $= K.\mathrm{a}.(\mathrm{t}) - \tau)$ (3.11)
- If K $= 1$, there exit S (t) follows the entrance with A delay constant (τ) there difference between the output and input is called trailing error and is equal to $a.\tau$
- If K $\neq 1$, SP(t) And (t) have not the same slope. They diverge.

➢ ***Impulse Response:***

The entrance East data by e (t) $= E_0\delta(t)$ In Laplace E (p) = E $_0$. There exit East data by :

$$S(p) = \frac{kE_0}{1+\tau P} \Rightarrow S(t) = \frac{kE_0}{\tau} \quad (3.12)$$

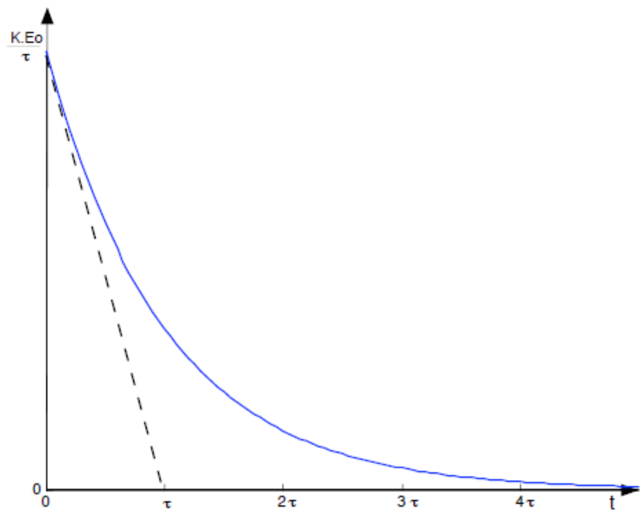


Figure 3.7: ***Impulse Response of a First-Order System***

3.2.2 Response of Second-Order Systems

3.2.2.1 Transfer Function

The equation differential more general of second order east:

$$b_0 s(t) + b_1 \frac{ds(t)}{dt} + b_2 \frac{d^2 s(t)}{dt^2} = a_0 e(t) + a_1 \frac{de(t)}{dt} + a_2 \frac{d^2 e(t)}{dt^2} \quad (3.13)$$

In this section, we will only study systems where the derivatives of the input do not appear ($a_2 = a_1 = 0$). The transfer function of these systems can be set in canonical form:

$$T(p) = \frac{k}{1+\frac{2\xi p}{\omega_n}+\frac{p^2}{\omega_n^2}} \quad (3.14)$$

With K, the static gain of the system.

ω_n The pulse natural (in rad/s). We can set $\tau_n = 1/\omega_n$. This same quantity is also called natural frequency and denoted ω_0.

ξ : The depreciation coefficient .

If on look for the poles of their function of transfer: $1 + \frac{2\xi p}{\omega_n} + \frac{p^2}{\omega_n^2} = 0$

$$\sqrt{\Delta'} = \omega_n\sqrt{\xi^2 - 1}$$

On distinguished three case possible :

- $\xi > 1$ In this cases poles are real :$-\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$
- $\xi = 1$ the two poles are equal and real. They worth $-\omega_n$
- $\xi < 1$ the two poles are of the complexes conjugated. They are has part real negative if $\xi > 0$.

3.2.2.2 Step Response for ξ > 1

On speak of system has strong amortization. The two poles real *p1* And *p2 give* a answer that will be the sum of two exponentials. For an entry $e(t) = E_0\, u(t) \rightarrow s(p) = \frac{E_0}{p}$

There exit East data by: $$s(p) = \frac{kE_0\omega^2}{P(P-P_1)(P-P_2)} \quad (3.15)$$

$$s(t) = KE_0\left[1 - \frac{\tau_1}{\tau_1-\tau_2}e^{-\frac{t}{\tau_1}} + \frac{\tau_2}{\tau_1-\tau_2}e^{-\frac{t}{\tau_2}}\right]u(t), \quad (3.16)$$

With $p_1 = -\frac{1}{\tau_1}$ and $p_2 = -\frac{1}{\tau_2}$

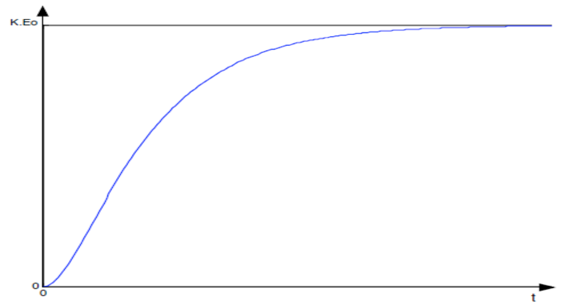


Figure 3.8: Step Response of an Overdamped Second-Order System

The characteristics of this answer are:

- The diet permanent East (t) = KE 0
- Has the origin, there tangent is horizontal

3.2.2.3 Step Response for ξ = 1

By report at paragraph previous, the poles are confused.

$$T(p) = \frac{k\omega_n^2}{(p+\omega_n)^2} \quad (3.17)$$

$$s\ (\text{t}) = \text{KE}_0\left[1 - (1+\omega_n t)\, e^{-\frac{t}{\tau_n}}\right]u(t), \quad (3.18)$$

There curve of answer looks has there curve obtained at paragraph previous, but Growth is faster.

3.2.2.4 Step Response for ξ < 1

On speak of system has weak amortization. The poles are complexes conjugated temporal East :

$$s\ (\text{t}) = \text{KE}_0\left[1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\, t + \varphi\right)\right] \quad (3.19)$$

With : $\varphi = \frac{\sqrt{1-\xi^2}}{\xi}$

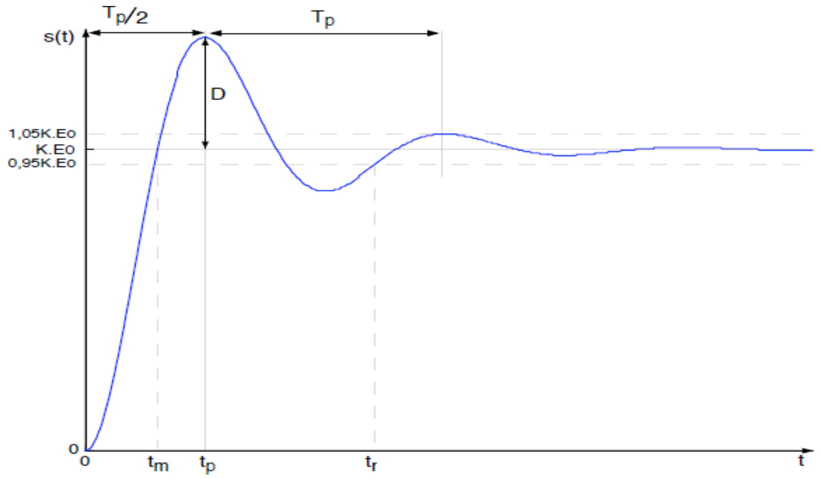


Figure 3.9: Step Response of an Underdamped Second-Order System

The characteristics of this answer are :

- Diet permanent s_p (t) = KE 0
- Has the origin, there tangent is horizontal
- Pulsation own amortized :

  $\omega_p = \omega_n\sqrt{\xi^2 - 1}$
- Pseudo-period of the oscillations : $T_p = \frac{2\pi}{\omega p}$
- Rise time (time at which *s(t)* first reaches *S* (t))

$$t_m = \frac{T_P}{2}\left(1 - \frac{\varphi}{\pi}\right) \quad (3.20)$$

- Time of peak $t_p = \frac{T_P}{2} = \frac{\pi}{\omega p}$
- The 5% response time is the time it takes for the output to reach steady state. has 5% close and y stay. The abacus attached given This time in function characteristics of the transfer function. an approximation for $\xi < 1$ *est*

$$t_r = 3\frac{\tau_n}{\xi} = \frac{3}{\xi\omega_n} \quad (3.21)$$

Who East the time of answer of the envelope exponential.

- The exceeding $D = S(t_p) - KE_0$ the calculation gives

$$D = K.E_0 e^{-\frac{2\pi}{\sqrt{1-\xi^2}}} \quad (3.22)$$

- On can Also define The exceeding relative (without unit) :

$$D_r = \frac{D}{K.E_0}. E_0 e^{-\frac{2\pi}{\sqrt{1-\xi^2}}} \quad (3.23)$$

- Exceeding successive: The report between two overruns successive of the same sign can be used to identify the damping ξ.

$$ln\frac{D_2}{D_1} = \frac{-2\xi\pi}{\sqrt{1-\xi^2}} \quad (3.24)$$

3.2.2.5 Response of a Second-Order System to a Ramp Input

The entrance East a ramp slope a,$E(p) = \frac{a}{p^2}$ on deduced there exit.

$$S(p) = \frac{ka}{p^2\,(p^2+2\xi\omega_n p+\omega_n{}^2)} \quad (3.25)$$

For $\xi > 1$ $$s(t) = K.a\left[t - \tau_1 - \tau_2 + \frac{\tau_1{}^2}{\tau_1-\tau_2}e^{-\frac{t}{\tau_1}} + \frac{\tau_2{}^2}{\tau_1-\tau_2}e^{-\frac{t}{\tau_2}}\right] \quad (3.26)$$

For $\xi < 1$ $$s(t) = K.a\left[t - \frac{2\xi}{\omega_n} + \frac{e^{-\frac{2\xi}{\tau n}}}{\omega_p}\sin(\omega_p\, t - \psi)\right] \quad (3.27)$$

With: $\psi = -2\arctan\frac{\sqrt{1-\xi^2}}{\xi}$

In the two case, the diet stationary East a right of slope *Ka* in the case $\xi < 1$ the diet transitional is oscillating.

3.2.3 Higher-Order Systems

Exact modeling of real physical processes often leads to transfer functions of order higher than two. The analysis of these systems is more complex, but methods and approximations do exist.

3.2.3.1 Difficulties in analysis

Unlike first- and second-order systems, there are no simple, universal formulas for predicting response time, overshoot, etc., for any given order. The time response results from the combination of several modes (exponential, oscillatory) related to the poles of the transfer function.

3.2.3.2 Study Methods

✓ **Analysis by Simulation**: This is the most common approach. Using software tools (MATLAB, Scilab , Python), the system's response to a step or ramp is simulated to directly visualize its performance.

✓ **Approximation Using a Simplified Model**: A widely used method in control theory involves approximating a higher-order system with a first- or second-order system. The idea is to identify the dominant poles. These are the poles (generally the slowest, those with the longest time constant) that most influence the transient response. The effect of faster poles can then be neglected if they are sufficiently far from the dominant poles.

- **Example** : A 3rd order system with poles at $P_1 = -1$, $P2$ = -10 and $P3$ = -10 will *have* an overall response mainly dictated by the slow pole p1 . It can often *be* approximated by a first order time constant τ = 1 second.

3.2.3.3 Considerations on Stability and Performance

For higher-order systems, the stability criterion remains the same: all poles of the transfer function must have a negative real part. However, the presence of multiple poles makes the system potentially more sensitive to certain parameter variations. Designing controllers for these systems is also more complex and requires specific methods (for example, the root locus method).

Noticed :

Understanding the response of first- and second-order systems is fundamental to the control of continuous systems. Their responses to typical signals allow us to define clear performance criteria such as speed (time constant τ) and accuracy (trailing error *e1*) . Although real-world systems are often higher-order, the concepts studied here remain essential, as they form the basis for their analysis and control, particularly through dominant-pole approximation .

3.3 System identification

The purpose of system identification is to determine the parameters of a mathematical model, such as a transfer function, based on the analysis of the experimentally measured time response. This approach is essential for designing appropriate controllers, particularly when the system under study does not have a known mathematical model a priori.

3.3.1. Identification of a first-order system

Assuming that the system under study is of the first order:

Equation (3.3) ⇒ $T(P) = \frac{k}{1+\tau p}$

3.3.1.1. Identification Method Using Step Response

Following the excitation of the system by a step, the identification of the parameters *k* and τ is carried out by the systematic analysis of its time response.

- ***Practical steps:***

1. Apply a step of amplitude E_0 and record the response *s (t)*.
2. Determine the static gain *k* :

$$k = \frac{s(\infty)}{E_0}$$

Or :

- *s(t)* **:** Represents the exiting the system in the time domain
- E_0 : Amplitude of the entry step
- *s(∞)* is the final value of the output

3. Determine the time constant τ

Different methodological approaches can be used to determine the time constant :

- ✓ **Graphical method at 63%:**

Identify the time when the output reaches 63% of its final value
At $t=\tau$, $s(\tau)=0.63.kE_0$

- ✓ **Method of finding the tangent at the origin:**

Draw the tangent to the curve at the origin
This tangent intersects the horizontal asymptote a: s = $k \times E_0$ at $t=\tau$.

- ✓ **5τ Method:**

The steady state is reached at t≈5τ

$s(5\tau) \approx 0.993.\ kE_0$

Example

- ✓ **Experimental data:**

- Input step: $E0 = 2v$
- Final output: $s(\infty) = 5v$
- Time to reach 63% of the final value: $t=1.2s$.

- ✓ **Identification:**

1. Static gain: $k = \frac{5}{2} = 2.5$.
2. Time constant: $\tau=1.2s$.

- ✓ **Identified model:**

$$T(p) = \frac{2.5}{1+1.2p}$$

✓ **Verification by different points**

Time(s)	% final value	Theoretical value
τ	63%	*2.5 × 2 × 0.63 = 3.15v*
τ	86%	2.5 × 2 × 0.86 = 4.3 *v*
3τ	95%	2.5 × 2 × 0.95 = 4.75 *v*

3.3.2 Identification of a second-order system

Assuming that the system under study is of the second order :

Equation (3.14)⇒ $T(p) = \frac{k}{1+\frac{2\xi p}{\omega_0}+\frac{p^2}{\omega_0^2}}$

3.3.2.1. Identification Method Using Step Response

We can follow a similar approach to that used for the first order system, but in the case of a second order system, we need to determine the parameters *k*, ζ, ω_0 and the first overshoot D_1, as well as the pseudo -period *Tp*.

We will only study the case where **(0 < ζ < 1)**

➢ ***Identification method:***

1- Determine the static gain *k* :

$$k = \frac{s(\infty)}{E_0}$$

2- Measure the first overshoot *D1* :

$$D_1 = s_{max} - s(\infty)$$

3- Calculation of the relative overrun: $D\% = \frac{D_1}{S(\infty)}.100.$

4- Determine ζ using the formula:

$$\zeta = \frac{-ln\left(\frac{D\%}{100}\right)}{\sqrt{\pi^2+ln^2\left(\frac{D\%}{100}\right)}}$$

5- Measuring the pseudo-period *Tp* :

Time between two successive peaks.

6- Calculate *ω0* :

$$\omega_0 = \frac{2\pi}{T_P\sqrt{1-\zeta^2}}$$

- **Example**

✓ **Experimental data:**

- Input step: *E0* = *3v*
- Final output: *s(∞)* = *6v*
- First overshoot *;* s_{max}=7.2V
- Pseudo-period: *Tp* =0.8 *s.*

✓ **Identification:**

1- $k = \frac{6}{3} = 2$

2- $D_1 = 7.2 - 6 = 1.2\ v$

3- $D\% = \frac{1.2}{6} 100 = 20\ \%$.

4- $\zeta = \frac{-ln(0.2)}{\sqrt{\pi^2 + ln^2(0.2)}} = \frac{1.069}{\sqrt{9.87+2.59}} \approx 0.456$

5- $\omega_0 = \frac{2\pi}{0.8\sqrt{1-0.456^2}} = \frac{6.283}{0.8\ .\ 0.89} \approx 8.82\ rad/s$

✓ **Identified model:**

$$T(p) = \frac{2}{1+\frac{2.0.456}{8.82}p+\frac{p^2}{8.82^2}} = \frac{2}{1+0.103\ p+0.0128\ p^2}$$

Similarly, a brief internet search can identify cases of second-order systems that we have not studied.

Remarks :

- Perform several trials to average the results.
- Use different input amplitudes to check linearity.
- Start with a simple (first order) identification and make it more complex if necessary.

In conclusion:

- In modeling, we start with knowledge of the (physical) system to obtain the model.
- In identification, we start from the system's response (measurements) to work our way back to the model.

Chapter 4: Frequency Response of Linear Systems

4.1Introduction :

Frequency response analysis involves studying the behavior of a system subjected to sinusoidal inputs of different frequencies. This response can be visualized using several graphical representations, such as Bode plots, Nyquist plots, and Nichols charts.

4.1.1 Bode Diagram:

The Bode plot allows us to study the frequency response of a system linear transfer function $G(p)$. To do this, we replace p by $j\omega$, which allows us to write the function of transfer below their shape next: $G(j\omega) = A(\omega)e^{j\varphi(\omega)}$. In this expression, we consider the module $A(\omega)$ And the argument $\varphi(\omega)$ of these function complex configured by the pulse ω. The diagram of Bode east got in tracing (asymptotically) the following functions of $A(\omega)$ and $\varphi(\omega)$ on logarithmic scales x-axisω (Figure 4.1)

$$A|_{dB} = 20log_{10}|G(j\omega)| \qquad \text{And} \qquad \varphi = arg\big(G(j\omega)\big) \qquad (4.1)$$

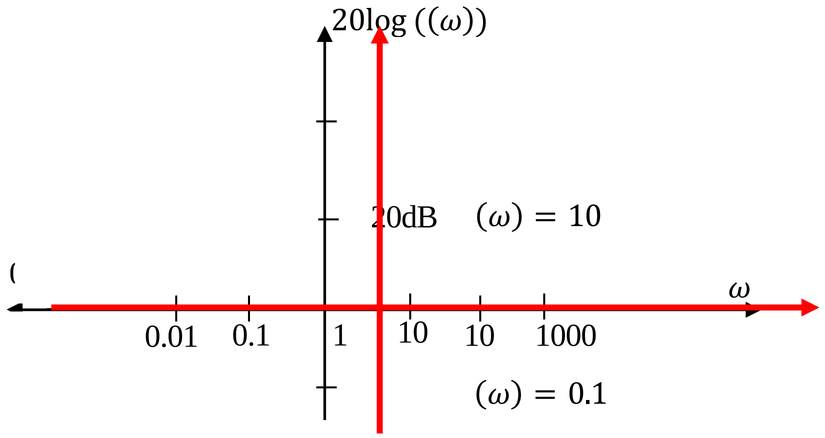


Figure 4.1 : Ladder logarithmic of diagram of Bode

It should be noted that one of the advantages of the Bode plot is that the overall diagram is the algebraic sum of the partial diagrams. We will give some examples of Bode plots.

- **Example 1**

We consider an open-loop transfer function system: $G(p) = p$ (4.2)

To study the frequency response of this system, we replace $p = j\omega$.,

Hence: $G(j\omega) = j\omega.$ (4.3)

To use Using Bode plots, we seek the expressions for the gain and phase of the transfer function.

The gain East ($G|_{dB} = 20log_{10}|j\omega|$4.4)

Hence: $G|_{dB} = 20log_{10}\omega$ (4.5)

It is this curve that must be approached asymptotically. It has a straight line with a slope equal to:

+20dB/decade (decade= [ω 10ω])

There phase is $\varphi = \arg\left(\frac{\omega}{0}\right)$= $\arg(\infty) = \frac{\pi}{2}$, it is constant($90^0$). (4.6)

There curve asymptotic East data by this figure 4.2.

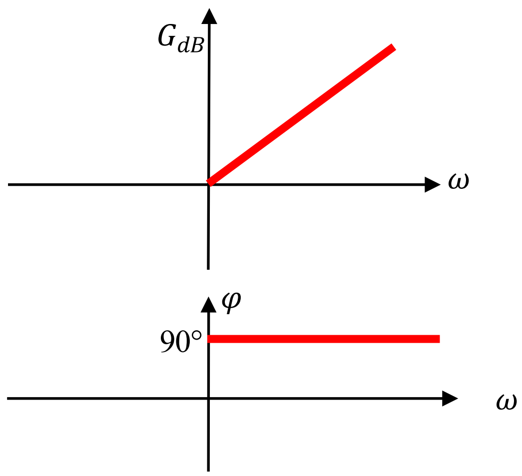


Figure 4.2: Diagram of Bode asymptotic of G(p)

Example 2

On considered the system of function of transfer:

$$G(p) = \frac{k}{1+\tau p} \quad (4.7)$$

East a constant positive (there constant of time).

$$G(j\omega) = \frac{k}{1+j\tau\omega} \quad (4.8)$$

The gain in dB:

$$G_{dB} = -20log_{10}(1+\tau^2\omega^2)^{\frac{1}{2}} = -10log_{10}(1+\tau^2\omega^2) \quad (4.9)$$

And there phase: $\varphi = -artan(\tau\omega)$ (4.10)

The asymptotic diagram is obtained by the following reasoning. We study the gain at low and high frequencies. Low and high frequencies are determined relative to a cutoff frequency. ω_c which is defined by $\tau\omega_c = 1$.

-- If so $\omega \ll \omega_c$, the low-frequency gain is : $G_{dB} = -10\log_{10}(1) = 0dB$. So in low frequencies: from $\omega = 0$, up to $\omega = \omega_c$ the gain in decibels is zero.

-- If $\omega \gg \omega_c$, the gain in high frequencies east : $G_{dB} = -20\log_{10}(\tau\omega)$. So in high frequencies: of $\omega = \omega_c$ until $\omega = \infty$, the gain is asymptotically linear.Of even, for the phase shift,

-- If $\omega \ll \omega_c$, the phase shift in bass frequencies East : $\varphi = -arg(0) = 0^0$, in low

Frequencies: from $\omega = 0$, up to $\omega = \omega_c$, The phase shift in decibel is practically null.

-- If $\omega \gg \omega_c$, the phase shift at high frequencies is : $\varphi = -arg(\infty) = -90^0$. So, at high frequencies: of $\omega = \omega_c$ until $\omega = \infty$, the phase shift East asymptotically equal at 90°.

Having determined the asymptotic behavior, it remains to specify the values of the gain and phase shift in the vicinity of the cutoff frequency. This is achieved by setting $\omega = \omega_c$. We can deduce the gain, which is -3 dB.

$$G_{dB}(\omega = \omega_c) = -20log_{10}(1 + \tau^2\omega^2)^{\frac{1}{2}} = 20log_{10}(2) = -3dB \qquad (4.11)$$

For the phase shift has there frequency of cut, on has :

$$\varphi = -arg(\tau\omega_c) = -arg(1) = -45^0 \qquad (4.12)$$

The Bode diagram the asymptotic value is given by the figure:

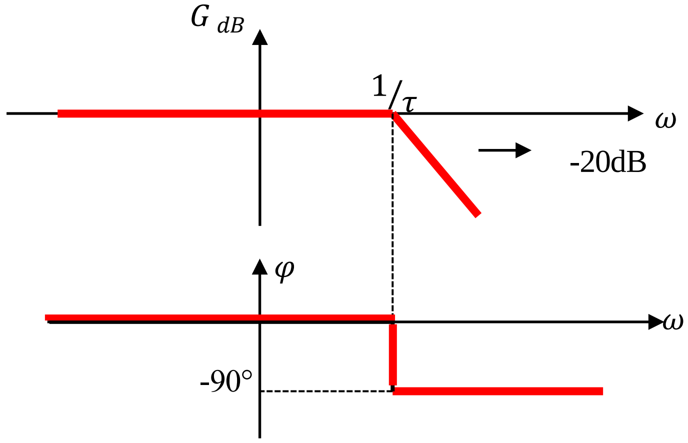


Figure 4.3: Diagram of Bode asymptotic of a system of first order

Example 3: ***System of second order***

Consider the system of transfer functions $G(p) = \frac{\omega_n^2}{p^2+2\xi\omega_n p+\omega_n^2}$ with $0< \xi<1$ (4.13)

On has Already determined the expressions of gain and of phase shift who are:

$$G(\omega) = \frac{1}{\sqrt{\left(1-\omega_n^2\right)^2 + 4\xi^2\omega_n^2}} \qquad \text{And} \qquad \Phi(\omega) = -\frac{2\xi\omega_n}{(1-\omega_n^2)} \qquad (4.14)$$

- Asymptotic behavior

The expression for the gain is: $G_{dB} = -20log_{10}\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2\right]^{\frac{1}{2}}$ (4.15)

--In bass frequencies,$\frac{\omega}{\omega_n} \ll 1.$ < $G_{dB} \cong -20log_{10}(1) \approx 0$. The asymptote in low frequency is zero.

- In high frequencies,$\frac{\omega}{\omega_n} \gg 1.$ < $G_{dB} \cong -40log_{10}\left(\frac{\omega}{\omega_n}\right)$. The asymptote in high frequencies is a slope of -40 dB /decade.

- If there pulsation of signal entry east ω_c, then $\frac{\omega_c}{\omega_n} = 1$:the gain is :

$$G = -20log_{10}[(2\xi)^2]^{\frac{1}{2}} \qquad (4.16)$$

This gain for the cutoff frequency depends on coefficient depreciation. If on assumed $\xi = 1$, the gain east of -6dB.

On carried out a reasoning similar For determine the diagram asymptotic phase shift.

$$\Phi(\omega) = -\arg\left(\frac{2\xi\omega_n}{(1-\omega_n^2)}\right) \qquad (4.17)$$

- In low frequencies, $\frac{\omega}{\omega_n} \ll 1$. $\Phi(\omega) = 0$, the asymptote in the lower frequencies is zero.

- In high frequencies, $\frac{\omega}{\omega_n} \gg 1. \Phi(\omega) = -\arg\left(\frac{2\xi\omega_x}{(1-\omega_x^2)}\right) \approx -arg\left(\frac{1}{\infty}\right) \approx -arg(0) = -180^0$

The asymptote in high frequencies East a right horizontal at -90°.

-- If the angular frequency of the input signal is ω_c, then $\frac{\omega_c}{\omega_n} = 1$; the phase shift is:

$$\Phi(\omega_c) = -\arg(\infty) = -90^0$$

- ***Example 4:***

Consider the following transfer function: $F(p) = \frac{1}{p^2+0.5\,p+1}$ the diagram of Bode (amplitude and phase) East given by figure 4.4.

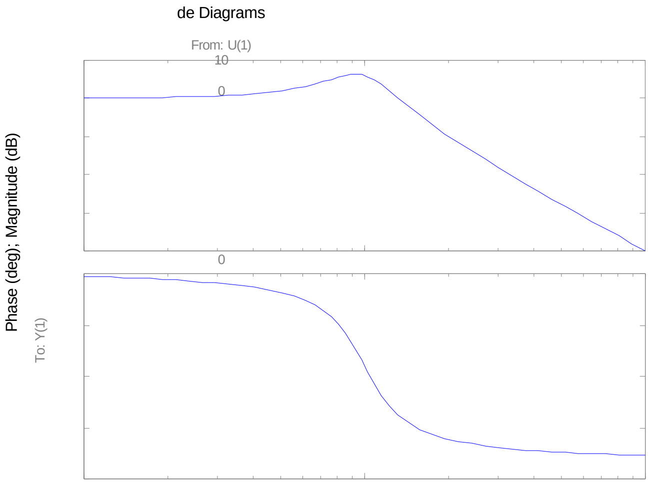


Figure 4.4: Diagrams de Bode of a system of second order

To the pulse of resonance is that who cuts the axis (0dB) has there pulse: $\omega_R = \omega_{Résonance}$

$$G|_{dB} = -20log_{10}\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2\right]^{\frac{1}{2}} \quad (4.18)$$

Drifting $G|_{dB}$ for $\omega = \omega_R$:

$$\frac{dG|_{dB}}{d\omega} = 0 \qquad \Rightarrow \quad \omega_R = \omega_n\sqrt{1-2\xi^2} \qquad \text{if } 2\xi^2 < 1 \ \ ou\ bien \ \ \xi < 0.7 \quad (4.19)$$

If ξ is very small: $\omega_R \approx \omega_n$ and

Case of $G|_{dB}Max = \frac{1}{1-\frac{\omega_R^2}{\omega_n^2}+2j\xi\frac{\omega_R}{\omega_n}}$ (4.20)

If < 0.7; module A passes through A Max, $G|_{dB}Max = \frac{1}{2\xi\sqrt{1-\xi^2}}$ (4.21)

This maximum is got by: $\omega = \omega_R = \omega_n\sqrt{1-\xi^2}$ (4.22)

The postman resonance or postman overvoltage: $Q = \frac{1}{2\xi\sqrt{1-\xi^2}}$ (4.23)

Diagram of Bode with Matlab

We can to use this function Bode of Matlab n giving directly the numerator and the denominator of the transfer function. For example:

$$F(p) = \frac{50}{p^3 + 9p^2 + 30\,p + 40}$$

The Bode instruction in Matlab is: Bode(50 , [[1 9 30 40]) given the diagram of Bode plot of the FT (amplitude and phase):

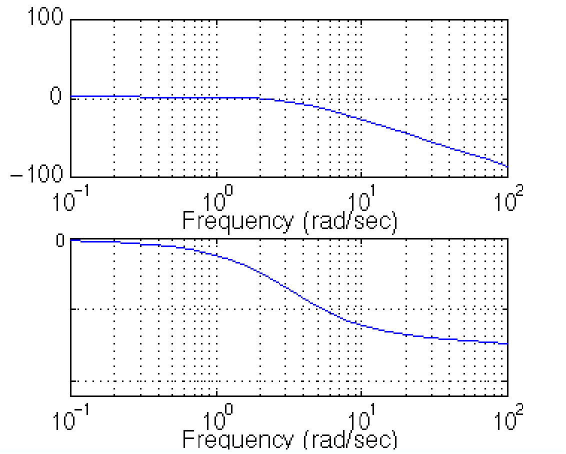


Figure 4.5: Diagrams of Bode for a system of order three.

There frequency East represented on a ladder logarithmic, there phase East data in degrees, and the amplitude in decibels.

4.1.2 Nyquist Plot

The Nyquist plot, or Nyquist diagram, is a parametric representation in the complex plane of the open-loop transfer function T (jω) when the angular frequency ω varies generally from $-\infty$ to$+\infty$. Each point of the locus corresponds to the image of T (jω) for a given angular frequency, the abscissa representing the real part and the ordinate the imaginary part.
Unlike Bode plots, which separate gain and phase, the Nyquist plot integrates these two aspects into a single representation, thus preserving the relationship between magnitude and argument
The Nyquist formulation is used to represent the harmonic response of a system given by its transfer function; it involves plotting the polar curve of the points M of contact details $G(\omega)$ and $\varphi(\omega)$when ω varies of 0 at $+\infty$. This curve is called the place Nyquist or polar location. (Figure 4.6)

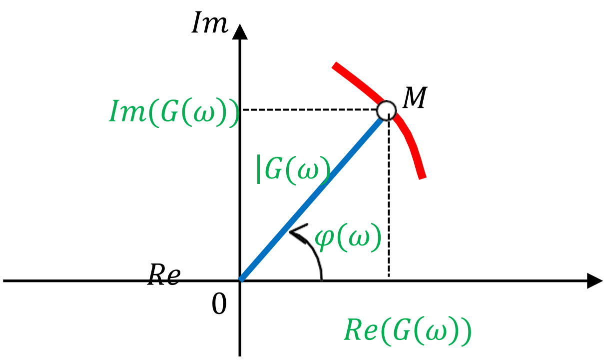


Figure 4.6: Definition of diagram of Nyquist

Example 1:

Either the system of function of transfer: $G(p) = \frac{1}{1+p}$

For establish the place by Nyquist, on replaces p by $j\omega$ in G(p) and we decompose G($j\omega$) into a real part and an imaginary part.

$$G(j\omega) = \frac{1}{1+j\omega} = \frac{1-j\omega}{1+j\omega} \quad \Rightarrow \quad Re(\omega) = \frac{1}{1+\omega^2} \quad \text{And} \quad Im(\omega) = \frac{\omega}{1+\omega^2}$$

The Nyquist locus is obtained for different values of $\omega$and from *0* to infinity. We are limited to an asymptotic study and the determination of a few notable points.

$$\omega \longrightarrow 0 \ \begin{cases} Re \longrightarrow 1 \\ Im \longrightarrow 0 \end{cases}: \omega \longrightarrow \infty \ \begin{cases} Re \longrightarrow 0 \\ Im \longrightarrow 0 \end{cases}: \omega \longrightarrow 1 \ \begin{cases} Re \longrightarrow 1/2 \\ Im \longrightarrow 1/2 \end{cases}$$

On obtains the trace of the place of Nyquist given by this figure (4.7)

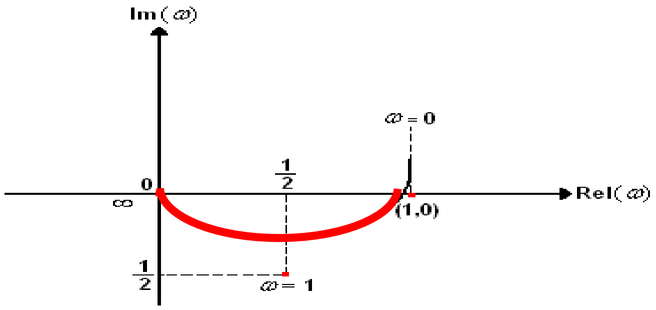


Figure 4.7: Trace of Nyquist of the FT G (p)

Example 2

Either the system function of transfer: $H(p) = \frac{1}{(p+1)(p+2)}$

To establish the Nyquist locus, we replace p with $j\omega$ in *H(p)* and decompose $H(j\omega)$ into a real part and an imaginary part as follows.

$$H(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)} = \frac{1}{2+j\omega-\omega^2+2j\omega} = \frac{2-\omega^2-3j\omega}{(2-\omega^2)^2+9\omega^2}$$

Hence: $Re(\omega) = \frac{2-\omega^2}{(2-\omega^2)^2+9\omega^2}$ and $Im(\omega) = \frac{-3j\omega}{(2-\omega^2)^2+9\omega^2}$

On determined a few points features for trace the place of Nyquist:

For: $\omega \longrightarrow 0$ we have: $Re(0) = \frac{1}{2}$ and $Im(0) = 0$

For: $\omega \longrightarrow \infty$ we have: $Re(\infty) = 0$ and $Im(\infty) = 0$

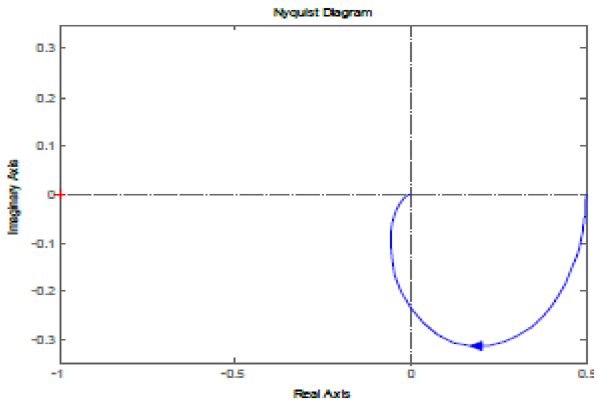


Figure 4.8 : Trace of Nyquist of the FT H (p)

4.1.3 *Black-Nichols Charts:*

There is a graphical tool that allows you to deduce the frequency response of the closed loop from the response open loop frequency is the Black-Nichols chart.

First, we need to obtain a functional closed loop (CL) diagram, called a unity feedback diagram, for which the gain of the feedback chain is equal to 1.

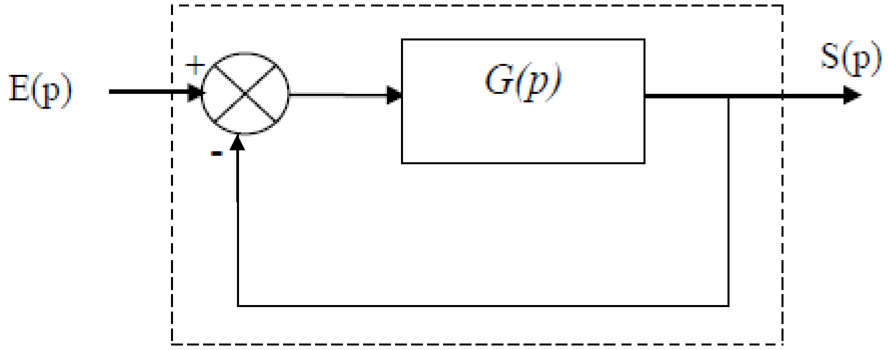


Figure 4.9: Functional Block of a loop closed has unit return

Function of loop transfer open (FTBO):

$$G(j\omega) = A(\omega)e^{j\varphi} = A(\omega)[cos\,\varphi + j\,sin\,\varphi] \quad (4.25)$$

Function of transfer to loop closed (FTBF) :

$$H\,(\mathrm{p}) = \frac{G(p))}{1+G(p))} \quad (4.26)$$

For

$$p = j\omega \;\Rightarrow\; |H(j\omega\,)| = \frac{|G(\omega\,)|}{|1+G(\omega\,)|} \quad (4.27)$$

$$|H(j\omega\,)| = \frac{A(\omega\,)}{|1+A(\omega)e^{j\varphi}|} = \frac{A(\omega\,)}{|1+A(\omega)[cos\,\varphi + j\,sin\,\varphi]|} \quad (4.28)$$

$$|H(j\omega)| = \frac{A(\omega)}{[(1+A(\omega)\cos\varphi)^2+(A(\omega)\sin\varphi)^2]^{1/2}} \quad (4.29)$$

$$|H(j\omega)| = \frac{A(\omega)}{[1+2A(\omega)\cos\varphi+A(\omega)^2]^{1/2}} \quad (4.30)$$

On look for now, there phase:

$$H(j\omega) = \frac{A(\omega)[\cos\varphi+j\sin\varphi])}{1+A(\omega)[\cos\varphi+j\sin\varphi])} \quad (4.31)$$

$$= \frac{A(\omega)[\cos\varphi+j\sin\varphi][1+A(\omega)[\cos\varphi-j\sin\varphi]])}{(1+A(\omega)\cos\varphi)^2+A(\omega)^2\sin\varphi^2} \quad (4.32)$$

$$\frac{Im}{reel} = \frac{\sin\varphi}{\cos\varphi+A(\omega)} \quad \Rightarrow$$

$$\langle H(\omega)\rangle = \arctan\frac{\sin\varphi}{\cos\varphi+A(\omega)} \quad (4.33)’$$

Abacus of Black is obtained by tracing (π, AdB) THE place such as

$$\begin{cases} 20logA(\omega) = Cte & \rightarrow \text{ Constante d'amplitude} \\ \varphi(\omega) == Cte & \rightarrow Constante\ de\ phase \end{cases}$$

- ✓ The curves will be defined between 0° and 360°.
- ✓ The curves must symmetrical beings by report $-\pi$

Noticed:

For the study of their stability there area who interests us East included between [-180°, 0dB]

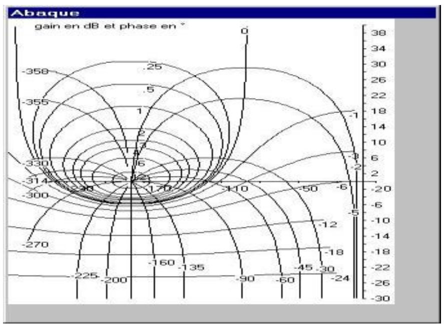


Figure**4.10: Abacus by Black Nichols**

Example 1:

Either the system function of transfer : $H(p) = \frac{1}{(p+1)(p+2)}$

Tracer rhe diagram of black Nichols by Matlab: Nichols ([1] , [132])

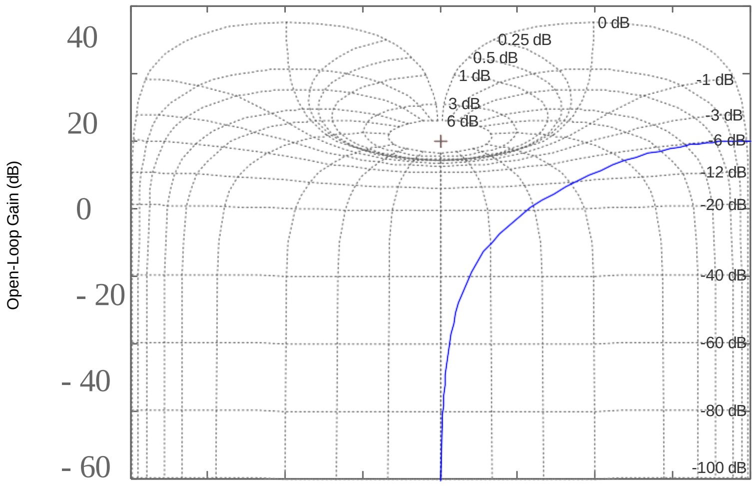


Figure 4.11 Black's Tracing Nichols of the FT H (p)

4.2 Stability Analysis and Criteria

4.2.1 Revers's Criterion in the Bode/Nyquist Plane

The revers's criterion is a simplified graphical rule for analyzing the stability of closed-loop control systems based on the frequency response of the open-loop transfer function. It is directly derived from the full Nyquist criterion, but its application is more straightforward under certain conditions.

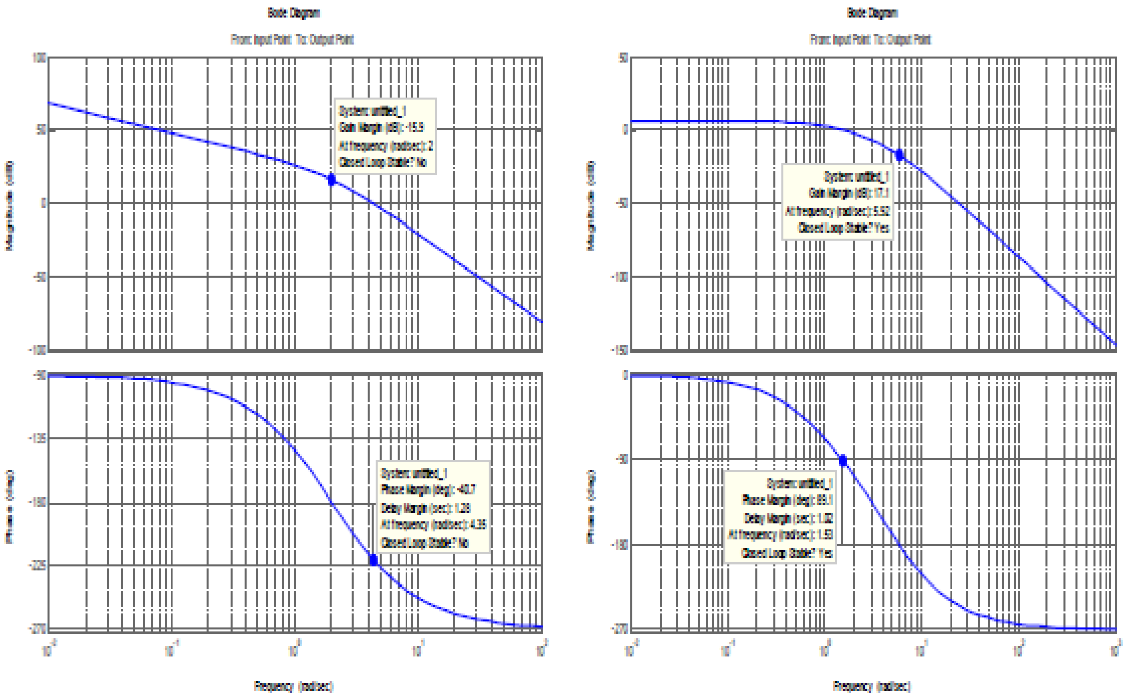
This criterion applies under a fundamental assumption: the open-loop transfer function must be stable and a minimum-phase system. This means it must have no poles and no zeros with a positive real part. If this condition is not met, the full Nyquist criterion must be used.

The general principle of the reverse criterion is to study the position of the frequency response relative to the critical point, which represents the stability limit. Here is its application in the two classical representation planes.

Representation Plan	Critical Point	Stability Rule
Nyquist Plan	Point (-1, 0) in the complex plane	Increasing pulsations ω.
Bode Plan	Gain = 0 dB at the -180° phase	For the angular frequency ωπ $\omega\pi$ where the phase is -180°, the magnitude (in dB) must be less than 0 dB.

4.2.1.1 ***Revers's Criterion from the Bode Diagram***

A system enslaved East stable if, has there pulse $\boldsymbol{\omega_1}$ for in which $|\boldsymbol{FTBO}|_{\boldsymbol{dB}}(\boldsymbol{j\omega}) = \boldsymbol{0dB}$**,** the phase shift is greater than ***-180°***



Unstable ***Stable***

Figure 4.12: Criteria of *Revers* in the diagrams of Bode

4.2.1.1.1 The margins of stability:

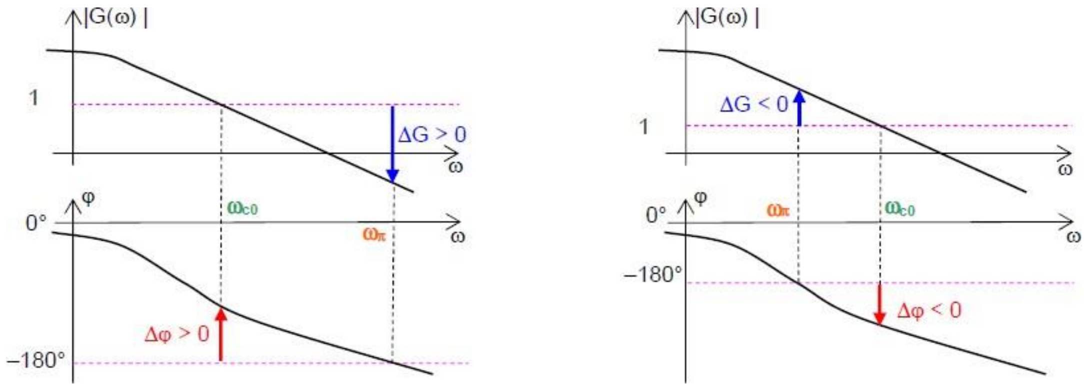
a) Margin of Gain (MG)

The pulse is determined, ω_c for which the phase shift is -180 °, the gain margin is the distance (in dB) between the curve and the x-axis.

b) Margin of Phase (MP)

On determined there pulsation ω_{c0} for which the gain East *0dB*, on measure their distance between the phase curve and *-180°*.

xample below : $\omega_{c0} = 0.5$; MP= 75°$\Delta\varphi$



System stable $\Delta\, G > 0, \Delta\, \varphi > 0$ *System unstable* $\Delta\, G < 0, \Delta\, \varphi < 0$

Figure 4.13: **Stability from Bode diagrams**

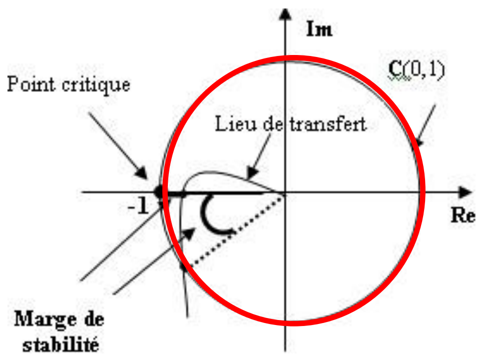
4.2.1.2 Determination of Stability Margins from the Nyquist Diagram

a) Margin of Gain (MG)

It is there distance between the point critical and the intersection of place of Nyquist with the axis real numbers. (Figure 4.15)

b) Margin of Phase (MP)

This is the angle between the negative real axis and the vector ***OM.*** Point ***M*** corresponds to the intersection of the transfer locus ***FTBO*** $(j\omega)$ with the circle centered at ***(0, 0)*** and with unit radius. The angular frequency at this point is ω_1. (Figure 4.14)



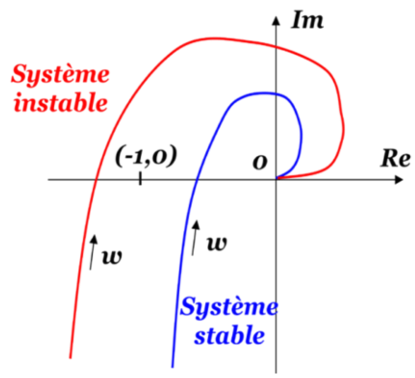


Figure 4.14 : Stability from Nyquist diagrams **Figure 4.15 : Reverse criterion from the Nyquist plot**

Noticed:

- A system that has a margin of Gain or a positive phase margin is a system stable.
- A system that has a negative gain margin or phase margin is a system unstable.
- A system that has zero gain margin or phase margin is a system at the limit of stability.

The values usual of the margins of stability allowing a correct setting of the control loops are:

Margin of Gain: 10dB has 15dB
Phase Margin: 40 ° to 45°
When the open-loop transfer function is explicitly known, conversely, the experimental plot is used.

4.2.2 Nyquist criterion

The Nyquist criterion is a powerful and general graphical method for the stability analysis of closed-loop systems. Formulated independently by Strecker (1930) and Nyquist (1932), it allows the determination of closed-loop stability from the open-loop frequency response, without requiring the explicit calculation of the poles of the closed-loop transfer function. For a closed-loop linear system to be stable, it is necessary and sufficient that the radius vector stretched between the point with affix -1 and the current point on the graph of the open-loop linear system's transfer function be true. $G(j\omega)$ Undergoes a phase shift exactly equal to - $l\,\pi$ where l is the multiplicity of the pole on the axis of the imaginaries.

- If the open loop transfer function $G(P)$has a zero pole with multiplicity , then the Nyquist diagram is discontinuous at $\omega = 0$. It must be assumed that the phase-shifting circuit describes L times a semicircle of infinite radius in a clockwise direction. By applying this rule, zero poles can be neglected; in other words, if there are no other unstable poles, the open-loop transfer function $G(P)$must be considered stable.
- If the open-loop transfer function $G(P)$is stable, then the closed-loop system is unstable for *any* cycle around the point -1.
- If the open-loop transfer function $G(P)$is unstable, then there must be a counterclockwise cycle around -1 for each pole of $G(P)$in the positive half-plane.
- The number of excess cycles ($N + P$ positive) is the number of unstable poles in the closed-loop system.
- However, if the curve intersects the point with affix -1, it is difficult to assert anything about the stability of the system, and the only conclusion that can be drawn from the diagram is that there are zeros on the $j\omega$.

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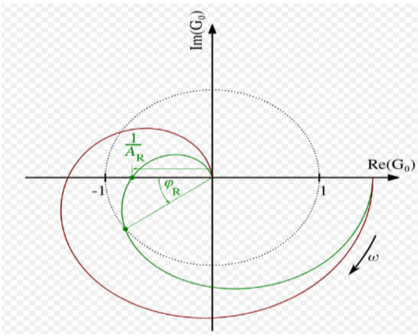

Red line: unstable
Green line: stable
subject to amplitude and phase conditions

Figure 4.16 Nyquist criterion

4.2.3 *Evans locus*

The root locus, also called the Evans locus, represents the location—in the complex plane—of the poles of the closed-loop transfer function (CLTF) as a function of the gain K. This occurs when the forward path gain varies from 0 to +∞. The Evans locus thus graphically represents, in the plane of S, the evolution of the poles of the closed-loop transfer function as the loop gain k varies from 0 to infinity. The closed-loop poles completely determine the system's stability, and examining their positions also allows us to determine the degree of stability. Therefore, it is of considerable interest to know the location of the poles of the closed-loop transfer function of a digital control system in the complex plane. The complex plane here is, of course, the plane of *P*.

4.2.3.1 Evans locus Construction Rules

The Evans locus is plotted from the position of the poles and zeros of the open loop and simple rules:

Rule 1: Determining the poles and zeros of the FT Where:

n = number of poles, m = number of zeros (with m ≤≥ n for a real system)

There is an asymptotic direction of nm (branches extend to infinity)

Rule 2: In the case where the coefficients of the characteristic equation are real, ⇒the Evans locus is symmetric with respect to the real axis.

Rule 3: The starting points of the locus (K = 0) are the poles of the open-loop transfer function. The arrival points of the locus (K → ∞) are the zeros of the open-loop transfer function.

Rule 4: Calculate the angle of the asymptotic direction with the real axis. The nm asymptotes of branches extending to infinity make the following angles with the real axis:

$$\varphi_i = \frac{\pm 180^0 (2i+1)}{n-m} \qquad (i = 0, 1, 2,\ldots\ldots) \qquad (4.34)$$

In the case where φ=π ⇒ The asymptote is therefore the real axis; there is no need to calculate the point of intersection of the asymptotes with the real axis.

Rule 5: Asymptotic branches meet, on the real axis, at the point with abscissa x_i such as

$$x_i = \frac{\sum pôles - \sum zéros}{n-m} \qquad (4.35)$$

Note: Any point on the real axis located to the left of an odd number of real poles and zeros is part of the locus.

Rule 6: The separation points ***μi*** (or meeting points) on the real axis can be calculated:

$$\sum \frac{1}{\mu - zéros} = \sum \frac{1}{\mu - pôles} \qquad (4.36)$$

Rule 7: (Intersection of the locus with the imaginary axis) The points of intersection of the locus with the imaginary axis can be easily determined either using Routh's stability criterion or by setting *P* by *j ω*in the characteristic equation, equating the imaginary and real parts of this equation to zero, and calculating ωand K. The pairs of values (ω, K) found give the frequencies and gains for which the locus intersects the imaginary axis.

4.2.4 Criteria algebraic of Routh:

The Routh criterion is a criterion that allows us to determine, from the characteristic polynomial (denominator of the closed-loop transfer function), the sign of the roots of this polynomial without solving the characteristic equation.

$$1+F.T.B.O=0 \quad (4.37)$$

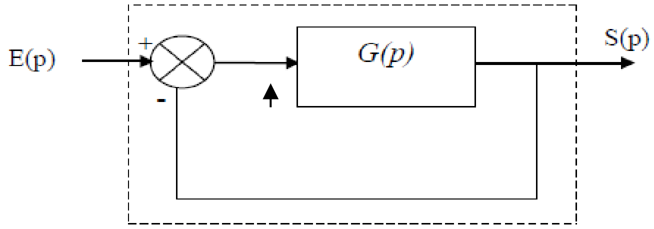

Figure 4.17: Closed-loop system

Noticed: In there following of course the FTBO East rated ***G(p).*** (figure4.17)
Either the equation characteristic of enslavement (Determiner) of the FTBF,

$$E(p)=1+F.T.B.O=1+G(p)=0 \quad (4.38)$$

We put it in polynomial form:

$$E(p)=1+a_n p^n+a_{n-1}p^{n-1}+\cdots+a_1 p+a_0=0 \quad (4.39)$$

Routh's table is constructed as follows:

$$\begin{array}{c|ccccc} p^n & a_n & a_{n-2} & a_{n-4} & \cdots \\ p^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\ p^{n-2} & b_m & b_{m-1} & b_{m-2} & \cdots \\ p^{n-3} & c_m & c_{m-1} & c_{m-2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \quad (4.40)$$

With:

$$\begin{aligned} b_m &= -\frac{1}{a_{n-1}}\det\begin{bmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{bmatrix} = \frac{a_{n-1}a_{n-2}-a_n a_{n-3}}{a_{n-1}} \\ b_{m-1} &= -\frac{1}{a_{n-1}}\det\begin{bmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{bmatrix} = \frac{a_{n-1}a_{n-4}-a_n a_{n-5}}{a_{n-1}} \\ &\vdots \end{aligned}$$

There procedure East iterated until p ^{0}

$$
\begin{aligned}
c_m &= -\frac{1}{b_m}\det\begin{bmatrix} a_{n-1} & a_{n-3} \\ b_m & b_{m-1} \end{bmatrix} = \frac{b_m a_{n-3} - b_{m-1}a_{n-1}}{b_m} \\
c_{m-1} &= -\frac{1}{b_m}\det\begin{bmatrix} a_{n-1} & a_{n-5} \\ b_m & b_{m-2} \end{bmatrix} = \frac{b_m a_{n-5} - b_{m-2}a_{n-1}}{b_m} \\
&\vdots
\end{aligned}
$$

Result: the system is stable if all the coefficients in the first column are of the same value. sign.

- The number of unstable poles (i.e., part (real positive) of the FTBF is equal to the number of sign changes on the first column.
- If during the calculation a zero is found in the first column only, it is replaced by $\varepsilon \ll 1$ And on continues the calculation. For the analysis of stability on do tender ε towards 0.
- Obtaining zeros on an entire line corresponds to a pair of conjugate imaginary poles.

Example 1: either the equation characteristic of enslavement:

$$E(\mathrm{p}) = \mathrm{p}^4 + 6\mathrm{p}^3 + 13\mathrm{p}^2 + 12\mathrm{p} + 4 = 0 \qquad (4.41)$$

The corresponding Routh table is as follows:

$$\begin{array}{r|ccc} p^4 & 1 & 13 & 4 \\ p^3 & 6 & 12 & 0 \\ p^2 & 11 & 4 & 0 \\ p & 9.82 & 0 & 0 \\ p^0 & 4 & 0 & 0 \end{array} \qquad (4.42)$$

There is no sign change in the first column Therefore, the closed-loop transfer function has no poles with a positive real part; consequently, the closed-loop system is stable.

Example 2: either the equation characteristic of enslavement

$$E(\mathrm{p}) = p^5 + 6p^4 + 12p^3 + 12p^2 + 11p + 6 = 0 \qquad (4.43)$$

Routh's table is as follows:

$$\begin{array}{r|ccc} p^5 & 1 & 12 & 11 \\ p^4 & 6 & 12 & 6 \\ p^3 & 10 & 10 & 0 \\ p^2 & 6 & 6 & 0 \\ p & 0 & 0 & 0 \\ p & 12 & 0 & 0 \\ p^0 & 6 & 0 & 0 \end{array} \qquad (4.44)$$

A line equal has zero implied 1 pair of poles imaginary pure. Either

$$E_1(P) = 6\mathrm{p}^2 + 6 \qquad (4.45)$$

And on replaces In there line next by:

$$\frac{dE_1(P)}{dp} = 12p \qquad (4.46)$$

And in constitutes there table of Routh.

There is no sign change in the coefficients of the first column, so the closed-loop transfer function has no poles with a positive real part, but it does have two purely imaginary poles. Are the roots of:

$$E_1(P)= 6p^{2} + 6 = 0 \Rightarrow p = \mp j \qquad (4.47)$$

Note: Routh's criterion is very useful when the polynomial coefficients are control system tuning parameters.

Example 3: either a system place In a loop of regulation with unit return (Figure 4.17)

$$G(p) = \frac{k}{p(p^2+p+3)} \qquad (4.48)$$

Has help of criteria of Routh, find there value of K, for which the system loop closed is stable.
The system is stable for K < 3

Note: The mathematical criterion and Routh's criterion are criteria for absolute stability; they do not allow us to specify the stability margins of the system. In other words, they do not indicate the degree of stability or instability.

Chapter 5: Synthesis specifications

5.1 Introduction :

The regulation of dynamic systems is a fundamental pillar of modern automation. A regulator, also called a controller, is a device (hardware or software) that generates a command intended to maintain or bring a system towards a desired behavior, despite the presence of disturbances and uncertainties in its parameters.

Considering a system represented by its transfer function G (p), a controller C (p) is designed to ensure that the closed-loop system satisfies a set of predefined specifications. The fundamental structure of a closed-loop system is represented by the classic block diagram where the controller compares the setpoint $y_e(t)$ to the measured output y(t) to generate the error ε(t) = $y_e(t)$ - y(t), which serves as the basis for calculating the control signal u(t).

The objective of this chapter is to methodically present the different approaches to regulator synthesis, from the formulation of specifications to practical sizing methods.

5.2 Design Specifications

5.2.1 Stability

Stability is a fundamental condition of any closed-loop system. A system is said to be stable if, after a limited perturbation, it returns to its equilibrium point or follows a bounded trajectory.

5.2.1.1 Stability criteria:

- ✓ BIBO (Bounded Input Bounded Output) stability
- ✓ Routh-Hurwitz algebraic criterion
- ✓ Nyquist geometric criterion
- ✓ Stability margin: defined by the gain margin (MG) and the phase margin (Mφ)

5.2.1.2 Typical requirements:

- ✓ Phase margin: 45° ± 60°
- ✓ Gain margin: 6 dB ± 12 dB

5.2.2 Speed

Speed characterizes the system's response rate to a change in setpoint.

5.2.2.1 Main indicators:

- ✓ Rise time (t_r): time to go from 10% to 90% of the final value
- ✓ Peak time (t_p): time to reach the first maximum
- ✓ Settlement time (t_s): time to remain within a band of ±2% or ±5% of the final value
- ✓ -3 dB bandwidth (ω_{-3}dB): frequency at which the gain drops by 3 dB

5.2.3 Accuracy

Accuracy assesses the system's ability to follow the setpoint in steady state.

5.2.3.1 Types of errors:

- ✓ Position error : ε_p = lim_{p→0} ε(p) for a step input
- ✓ Speed error: ε_v = lim_{p→0} ε(p) for a ramp input
- ✓ Acceleration error: ε_a = lim_{p→0} ε(p) for a parabolic input

5.2.4 Classification of systems:

- ✓ Type 0: finite error for step, infinite for ramp
- ✓ Type 1: zero error for step, finite for ramp
- ✓ Type 2: zero error for step and ramp, finite error for parabola

5.3 Different Regulatory Structures

5.3.1 General principle of system correction:

The idea is to introduce into the direct chain, upstream of the system A (P), an additional transfer function device C (p), called a corrector, whose essential role should be to modify the performance of the initial system.

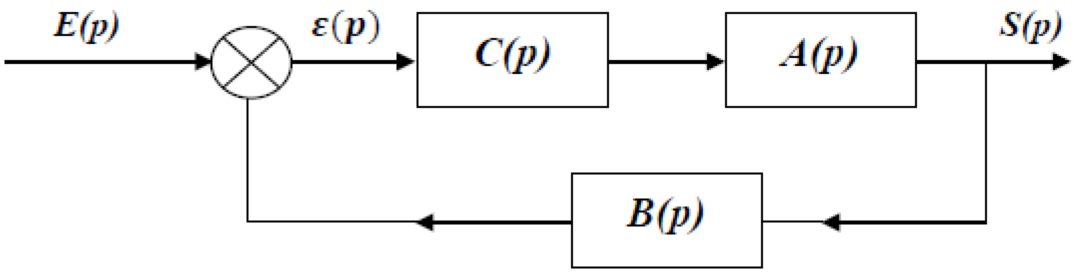

Figure 5.1: General diagram of a control loop

The whole art of system correction consists of choosing the right transfer function C (P) for this corrector so as to adjust each performance to its required value without disturbing, of course, the operation of the system.

***5.3.1.1* Elementary corrective actions (P, I, D):**

There are three basic corrective actions that can individually address specific performance issues. They are relatively simple to implement, but generally degrade other performance levels, making them suitable for situations with less demanding specifications. Otherwise, combining these actions within a more complex corrective mechanism should be considered.

5.3.1.1.1Proportional controller

The corrector is a simple adjustable gain amplifier C (P) =K whose purpose is to modify the initial static gain of the system.

• If $K<1$ in other words, it is an attenuator, the system's stability is improved and its overshoot in closed loop is reduced. On the other hand, speed and accuracy are degraded.

• If $K>1$ we improve the speed and accuracy of the closed-loop system but we decrease the stability, and we increase its overshoot.

5.3.1.1.2 Integral controller

The controller is a transfer function integrator: C (P) = 1 */p*, whose purpose is to add a zero pole to the open-loop transfer function (We already know that a system whose open-loop transfer function has a zero pole will be characterized by a zero position error).
The integral action controller is therefore intended to improve the accuracy of the servo system.

- ***Influence on other performance:***

The changes made to the transfer function undoubtedly alter the other performance aspects of the system.
Consider any open-loop transfer function system $G_i(p)$, the graphs represent respectively:

$$G_{idB} = 20log\ G_i(\omega) \quad (5.1)$$

$$\varphi_i(\omega) = argG_i(j\omega) \quad (5.2)$$

And the graphs corresponding to the corrected transfer function can be easily deduced from the initial graphs (Figure 5.2).

$$G_{cdB} = 20logG_C(\omega) = 20\ log\frac{G_i(\omega)}{\omega} = 20\ log\ G_i(\omega) - 20\ log(\omega) \quad (5.3)$$

$$\varphi_c\ (\omega) = argG_C(\omega) = arg\frac{G_i(\omega)}{j\omega} = \varphi_i(\omega) - \frac{\pi}{2} \quad (5.4)$$

We therefore move from the initial gain curveG_{idB} to the corrected curve G_{cdB} by "subtracting" from each segment the equivalent of a slope segment [1], in other words by decrementing each initial slope by one unit, at the angular frequency ω = 10, the gain dropped by 20dB. The phase diagram, meanwhile, is translated from $\frac{\pi}{2}$ down.

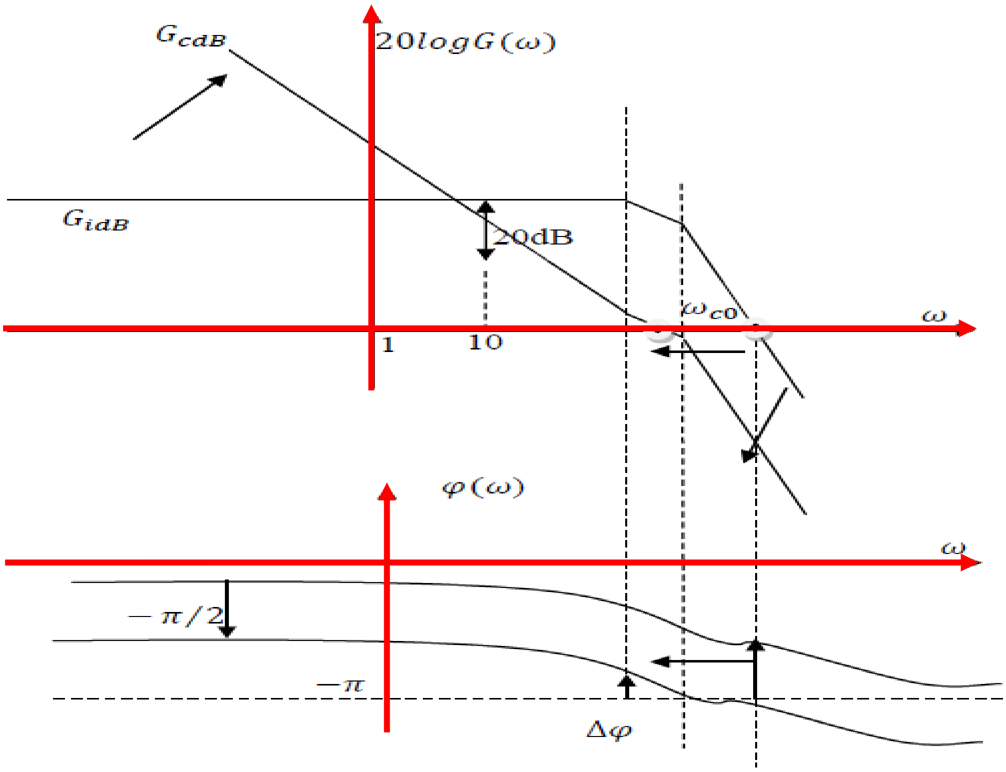

Figure 5.2 Influence of an integrator on performance

We observe that the cutoff frequency at 0dB decreases given that:

$$t_m \approx \frac{3}{\omega_{c0}} \tag{5.5}$$

We can deduce that the rise time increases, so the integrator will tend to slow down the closed-loop system.

Furthermore, the phase margin decreases, and the stability and limitation of overshoot are degraded.

In conclusion, only the accuracy of the system is improved by the introduction of an integral action corrector; all other performance aspects are diminished.

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5.3.1.1.3 Derivative controller

The controller is a transfer function differentiator

$$C(p) = p \tag{5.6}$$

Its objective is to add a zero to the open-loop transfer function. The action of this controller is the inverse of that of the integrator.

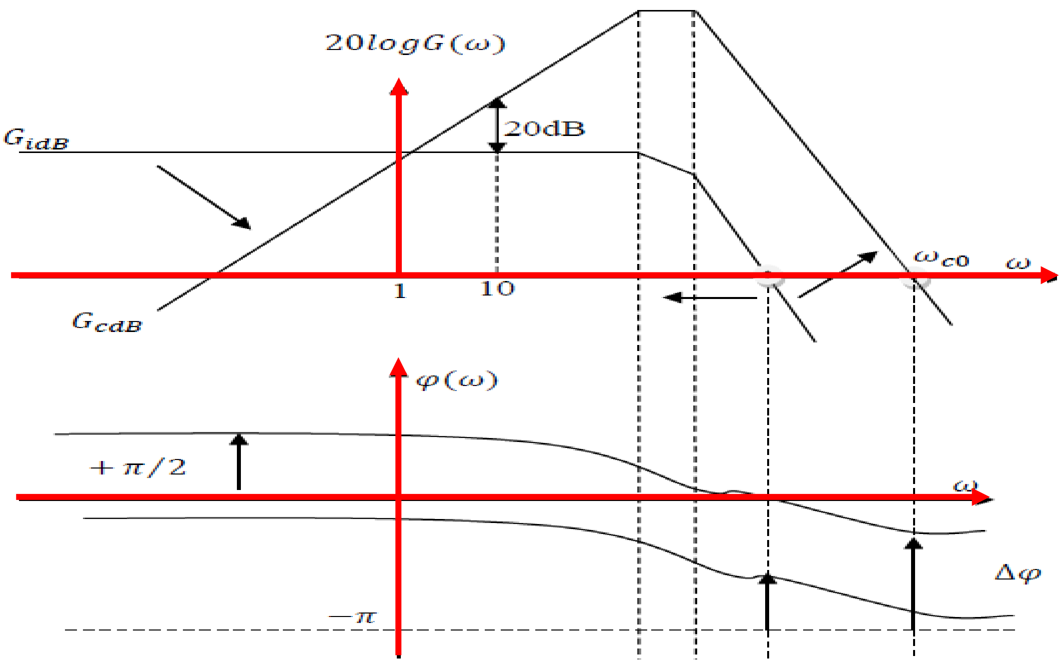


Figure 5.3: Influence of a derivatizer on performance

Consider again an arbitrary system of open-loop transfer functions $G_d(p)$, the graphs represent respectively:

$$G_{ddB} = 20log\ G_i(\omega) \tag{5.7}$$

$$\varphi_d(\omega) = argG_i(j\omega) \tag{5.8}$$

And the graphs corresponding to the corrected transfer function can be easily deduced from the initial graphs (Figure 5.3)

$$G_{cdB} = 20logG_C(\omega) = 20\ log\big(\omega G_d(\omega)\big) = 20\ log\ G_d(\omega) + 20\ log(\omega) \tag{5.9}$$

$$\varphi_c\ (\omega) = argG_C(\omega) = arg\big(\omega G_d(\omega)\big) = \ \varphi_d(\omega) + \frac{\pi}{2} \tag{5.10}$$

We therefore move from the initial gain curveG_{ddB} to the corrected curveG_{cdB} By "adding" to each segment the equivalent of a segment with a slope of 1, in other words by incrementing each initial slope by one unit, at the frequency ω = 10, the gain increased by 20dB. The phase diagram, meanwhile, is translated by $\frac{\pi}{2}$upwards.

We observe that the 0dB cutoff frequency increases, given that $t_m \approx \frac{3}{\omega_{co}}$

We can deduce that the rise time decreases, so the diverter will tend to accelerate the closed-loop system.

Increasing the ω_{c0}value also affects the phase margin, but this influence depends on the order of the system (it can make the system unstable).

The accuracy of the system, linked to the static gain, will be degraded by the derivative action since the low-frequency gain decreases sharply; in conclusion, only the speed of the system is improved by the introduction of a derivative-action corrector.

5.3.1.1.3 Proportional-Integral Controller (Phase-Lag)

A phase-delay controller, contrary to what its name might suggest, increases gain only at low frequencies. It is therefore used to improve the accuracy of a closed-loop control system.

Its transfer function is:

$$C(P)=\frac{\alpha(1+Tp)}{1+\alpha Tp} \qquad \text{with} \alpha > 1 \qquad (5.11)$$

To better understand the action of this corrector, let's plot its Bode diagram. There are two cutoff frequencies. $\frac{1}{T}$ and $\frac{1}{\alpha T}$

Such as: $\frac{1}{T} < \frac{1}{\alpha T}$

We have $$C(\omega)=\frac{\alpha\sqrt{1+T^2\omega^2}}{\sqrt{1+\alpha^2T^2\omega^2}} \qquad (5.12)$$

And $$\varphi(\omega) = \arctan(T\omega) - arctan(\alpha T\omega) \qquad (5.13)$$

When $$\omega \longrightarrow 0 \ , \quad \Longrightarrow \quad C(\omega) \ \longrightarrow \ \alpha \qquad (5.14)$$

When $$\omega \longrightarrow \infty \ , \quad \Longrightarrow \quad 20\log(C(\omega)) \ \longrightarrow \ 0dB \qquad (5.15)$$

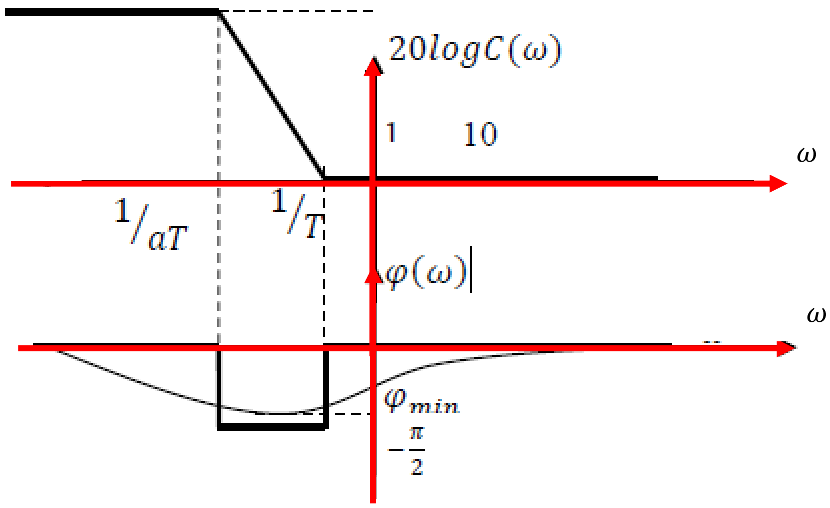


Figure 5.4: Bode plot of a phase-lag compensator

Examining the Bode plot allows us to predict the effect of this controller. When it is cascaded with the system to be corrected, in the direct chain the two Bode plots will add together, increasing the static gain, which $20log(\alpha)$improves accuracy. By setting the parameter T to a sufficiently low value, this correction only affects low frequencies; the high-frequency gain is practically unaffected. The additional negative phase shift introduced by the controller is also located at low frequencies. Therefore, it has no impact on the stability margin.

To adjust the phase-delay corrector, we will choose the value ofα which allows us to obtain the desired resulting static gain, and we will then choose T so that$\frac{1}{T} \ll \omega_{C0}$

5.3.1.1.5 *Proportional-Derivative Controller (Phase-Lead)*

The phase-lead corrector, as its name suggests, is a corrector which increases the phase margin of a system; it compensates for a phase shift that is too small around the *0dB cutoff frequency* .

We take:

$$C(P)=\frac{(1+\alpha Tp)}{1+Tp} \qquad \text{with} \alpha > 1 \tag{5.16}$$

To better understand the action of this corrector, let's plot its Bode diagram. There are two cutoff pulses $\frac{1}{T}$ and $\frac{1}{\alpha T}$

Such as: $$\frac{1}{\alpha T} < \frac{1}{T} \tag{5.17}$$

We have $$C(\omega)=\frac{\sqrt{1+\alpha^2T^2\omega^2}}{\sqrt{1+T^2\omega^2}} \tag{5.18}$$

And $$\varphi(\omega) = \arctan(\alpha T\omega) - \arctan(T\omega) \tag{5.19}$$

When $$\omega \longrightarrow 0 \ , \quad \Rightarrow \quad C(\omega) \longrightarrow 1 \tag{5.20}$$

When $$\omega \longrightarrow \infty \ , \quad \Rightarrow \quad 20\log\big(C(\omega)\big) \longrightarrow 20\log(\alpha) \tag{5.21}$$

The advantage of this corrector is visible on its phase diagram: at the angular frequency

$$\omega_{Max} = \frac{1}{T\sqrt{\alpha}} \tag{5.22}$$

This presents a maximum that we can easily calculate:

$$\varphi_{Max} = \arcsin\frac{\alpha-1}{\alpha+1} \tag{5.23}$$

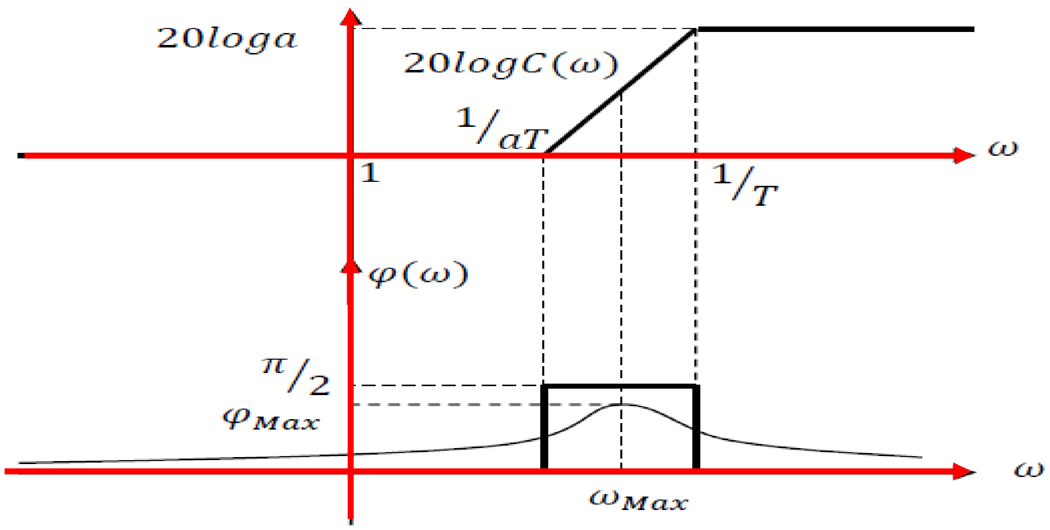


Figure 5.5: Bode plot of a phase-lead controller

The principle of corrective action consists of making coincide ω_{Max} with the 0dB cutoff frequency ω_{C0} of the system to be corrected and adjusted, φ_{Max} which is called phase rise, in order to obtain the desired phase margin.

5.3.1.1.6 PID controller

PID ⟹controller *PI* effect + *PD* effect

$$G_{PID}(P) = k + \frac{1}{\tau_i\, p} + \tau_d\, p = \frac{(1+T_1P)((1+T_2P))}{\tau_i\, p} \tag{5.24}$$

)

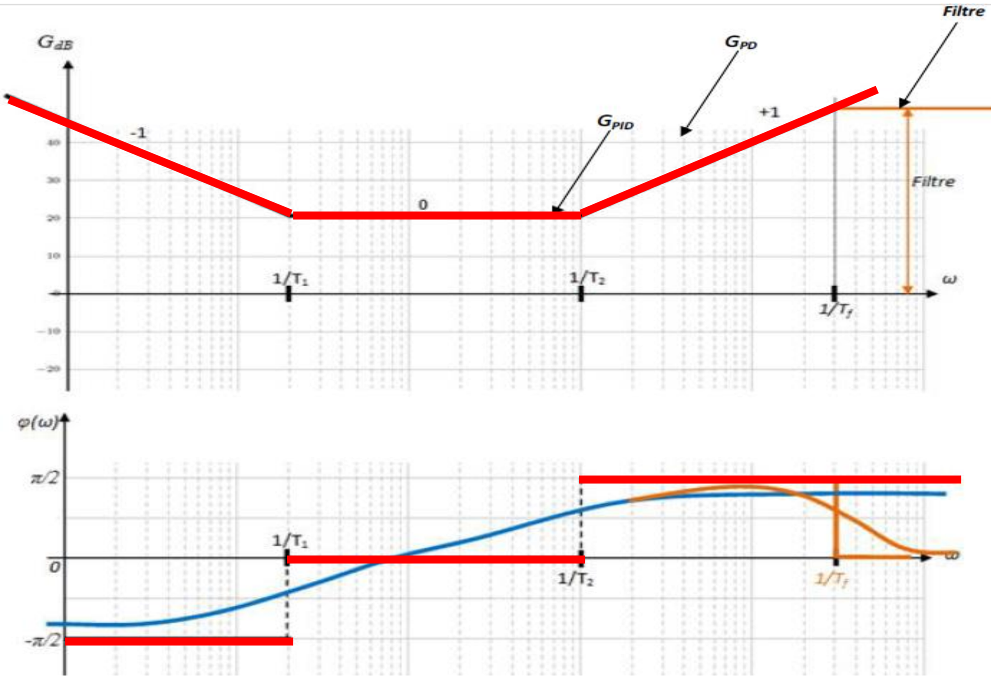


Figure 5.6: PI effect and PD effect

5.3.2 RST Regulator

The RST controller is an advanced control architecture with three degrees of freedom, enabling closed-loop control of industrial systems. This polynomial structure represents the most general formulation of linear controllers. In the single-variable framework, RST controllers offer remarkable operational flexibility, explaining their widespread adoption in digital control systems. Their implementation allows for independent specification of tracking, regulation, and disturbance rejection performance. However, extending this to the multivariable case presents significant analytical and practical complexities. In this context, the polynomial formalism of the RST reveals its limitations, making the adoption of a state-space representation preferable for the synthesis of multivariable controllers. This alternative approach offers better control of the interactions between variables and simplifies the design of control laws for complex systems.

5.4 Synthesis using empirical methods

The synthesis of controllers using empirical methods is a common approach in automatic control when a system model is unavailable or insufficiently accurate. These methods allow the parameters of a controller (usually a PID controller) to be determined from observations of the system's behavior. Among the best-known methods are the Ziegler-Nichols method, the Flat method (or "Relay Feedback"), the symmetric method, etc.

5.4.1 ZIEGLER-NICHOLS method

PI and *PID* controllers are among the most widely used analog controllers. The main problem lies in determining the controller coefficients *Kp, τi, and τd* . Several experimental methods have been developed to determine these coefficients. The method developed by Ziegler and Nichols is only applicable if the system under study can tolerate overshoot. The Ziegler-Nichols method consists of determining the critical gain KL at the stability limit (marginally stable system) in a closed loop. It requires looping the system through a simple proportional controller whose gain is increased until the system oscillates continuously (Figure 5.7); this is the system's stability limit. After recording the critical gain KL and the oscillation frequency ω_C of the response, the parameters of the chosen controller can be calculated using the table below. The values proposed by Ziegler and Nichols have been tested in a very large number of situations and it should be emphasized that here too they lead to a relatively short rise time coupled with a high overshoot.

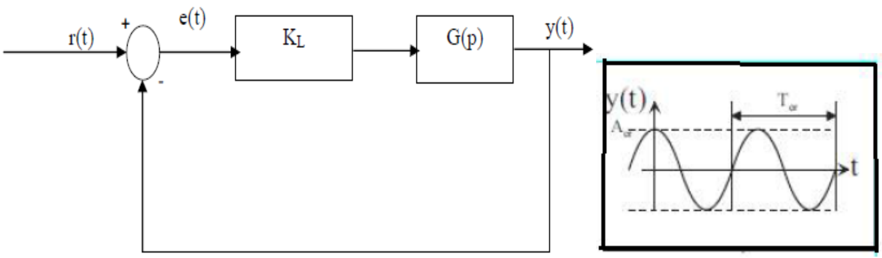


Figure 5.7: A simple marginally stable proportional regulator

Tableau 5.1: The regulator parameters can be calculated from the following table:

Regulator	K	τi	Td
P	0.5 K_L		
PI	0.45 K_L	$2\pi/(1.2\,\omega_C)$	
ID	0.6 K_L	π/ω_C	$\pi/(4\omega_C)$

ω_C : is the frequency of the oscillations of the marginally stable system.

5.4.2 Flat Mechanism (Relay Feedback)

The flat-field method, also called the relay feedback method, is an alternative to the closed-loop Ziegler-Nichols method. It allows the critical parameters *Ku* and *Tu to be determined* without having to manually adjust a proportional gain.

5.4.2.1. Principle

The controller is replaced with a relay with hysteresis (to avoid high-frequency oscillations due to noise). The system enters a self-sustaining oscillation. The following measurements are then taken:

- *A* : the amplitude of the oscillations
- τ_u: the period of the oscillations

5.4.2.2. Calculation of critical parameters

The critical gain can be estimated by:

$$K_u = \frac{4d}{\pi A} \tag{5.26}$$

Where *d* is the amplitude of the relay.

Next, we use the same formulas as Ziegler-Nichols in closed loop to tune the PID.

5.4.3 Symmetrical Method

The symmetric method (or double integrator method) is a tuning method that aims to obtain a closed-loop response that is symmetrical with respect to the setpoint. It is particularly well suited to systems exhibiting integrative behavior.

5.4.3.1 Principle

The controller is adjusted so that the closed-loop response has a symmetrical shape. Typically, an equal rise time and fall time are defined.

5.4.3.2 Adjustment rules

For a PID controller, the following formulas can be used (for example):

- ✓ $K_p = \frac{1}{2k\tau_d}$
- ✓ $\tau_i = 2\,\tau_d$
- ✓ $\tau_d = \frac{\tau_u}{4}$

Where τ_u is the desired oscillation period.

5.4.3.3. Comparative analysis and selection criteria

5.2:Comparative Table of Methods

Method	Complexity	Robustness	Precision	Scope of application
Ziegler-Nichols	Weak	Average	Average	Standard systems
Flat	Average	Good	Good	Aperiodic or oscillating response
Symmetrical	Weak	Good	Average	Symmetrical systems

5.5 Synthesis using graphical methods

Graphical methods for controller design represent a powerful approach in linear control, enabling the design of controllers based on frequency or time-domain representations of systems. Unlike empirical methods, they offer a solid theoretical foundation and allow for the achievement of specific performance levels.

5.5.1 Evans Root Locus Method

5.5.1.1 Principle

The Evans locus represents the evolution of the poles of the closed-loop transfer function as the gain varies from 0 to +∞. Synthesis consists of modifying this locus to satisfy temporal specifications.

5.5.1.2 Characteristic equations:

- ✓ Phase condition: $\angle G(p)\, H(p) = \pm 180°\,(2k+1)$
- ✓ Modulus condition : $|G(p)\, H(p)| = 1$

Where: $G(s)H(s)$ represents the open-loop transfer function.

5.5.1.3 Analysis of the uncorrected system

- ✓ Plot the initial Evans location
- ✓ Identify the dominant poles
- ✓ Evaluate performance (overshoot, response time)

5.5.1.4 Closed-loop specifications

- ✓ Eligible region in the complex plan
- ✓ Desired damping ratio ζ
- ✓ Natural pulse ω_n

5.5.1.5 Types of spell checkers by Evans location

1. **Phase-lead corrector**

- Structure : $C(P)$= k. $\frac{(1+\tau p)}{1+\alpha\tau p}$ with $\alpha < 1$
- Effect: Moves the location to the left → improves stability
- Application: When the desired poles are to the left of the initial location

2. Phase-delay corrector

- Structure : $C(P)$= k. $\frac{(1+\tau p)}{1+\beta\tau p}$ with $\beta > 1$
- Effect: Does not affect the high-frequency response but increases the low-frequency gain
- Application: Improving static accuracy

3. PID Controller

- Structure: C(s) = Kp + K_i/p + K_d p

5.5.1.6 Interpretation of Evans' location:

- ✓ Action P: movement along the location
- ✓ Action I: Adding a pole to the origin
- ✓ Action D: Add a zero

5.5.1.7 Design methodology

1. Position the poles and zeros of the corrector to attract the location towards the desired region.
2. Determine the gain from the modulus condition at the desired point
3. Verify that all closed-loop poles are within the permissible range

5.5.2. Bode plot method

5.5.2.1 Frequency Approach

Bode's method allows the synthesis of controllers from frequency specifications (phase margin, bandwidth, disturbance rejection).

5.5.2.2 Standard correctors in the Bode plane

1. Phase-lead compensator

- Objective: Increase the phase margin
- Setting :
  - $\alpha = (1 - \sin(\varphi_m))/(1 + \sin(\varphi_m))$. Where φ_m is the maximum phase lead
  - $\omega_m = 1/(\tau\sqrt{\alpha})$ positioned at the cutoff frequency
  - Maximum gain: $1/\sqrt{\alpha}$

Procedure:

1. Determine the required phase margin
2. Calculate α and the phase lead φ_m
3. Position ω_m at the new cutoff pulse
4. Adjust the gain to compensate for the 1/α attenuation

2. Phase-delay corrector

- Objective: To improve accuracy without compromising stability
- Setting :
  - β > 1 (typically 5 to 20)
  - Zero is placed one decade below the cutoff pulse

Procedure:

1. Determine the necessary gain increase at low frequencies
2. Choose β to obtain this gain
3. Position the zero at $\omega_2 = 1/\tau$ well below ω_c
4. The pole at $\omega_1 = 1/(\beta\tau)$

3. PID controller in the Bode plane

- Action P: Vertical translation of the gain diagram
- Action I: Additiona l -20 dB/decline slope in low frequency
- Action D: Additional +20 dB/decline slope in high frequency

5.5.3 *Use of Black-Nichols charts*

Black-Nichols nomograms allow direct synthesis from specified closed-loop performances.

5.5.3.1 *Design Procedure*

1. Plot the uncorrected open-loop transfer locus
2. Overlay the Black-Nichols abacus
3. Identify the position relative to the desired iso-M curves
4. Design the corrector so that the corrected locus follows the desired iso-M curve
5. Verify that the closed-loop specifications are met.

5.5.3.2 Specific advantages

- Direct visualization of closed-loop performance
- Optimizing the speed/stability trade-off
- Adaptation to the specifications in pursuit and regulation

5.5.4 Nyquist Place Method

The method uses the Nyquist criterion to ensure stability while meeting performance specifications.

5.5.4.1 Synthesis Procedure

1. Plot the Nyquist locus of the uncorrected system
2. Identify violations of constraints (proximity to point -1)
3. Design the corrector to move the location away from critical regions
4. Verify compliance with the Nyquist criterion

5.5.4.2 Geometric Constraints

- **Circle of stability** : Region around point -1 to be avoided
- **Contour of M** : Region corresponding to a maximum closed-loop gain
- **Contour of φ** : Region for the phase margin

5.6 Comparison and areas of application

Method	Benefits	Limitations	Typical applications
Evans's Place	Direct visualization of the poles, precise design	Limited to systems with concentrated parameters	Servomechanisms, position control
Bode	Simple, intuitive, good performance compromise	Less accurate for delay systems	Industrial process regulation
Black-Nichols	Closed-loop direct optimization	Less used today	Systems with strict closed-loop specifications
Nyquist	Delay handling, robustness	Complex to interpret	Delay systems, robustness analysis

The following table presents the Laplace transforms of certain elementary functions :

	f(t)	***F(p) Transformée de Laplace de f(t)***
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{p}$
3	$tu(t)$	$\frac{1}{p2}$
4	$t^n u(t)$	$\frac{n!}{pn+1}$
5	$e^{-at}u(t)$	$\frac{1}{p+a}$
6	$t^n e^{-at}u(t)$	$\frac{n!}{(p+a)^{n+1}}$
7	$Sin(\omega t)u(t)$	$\frac{\omega}{p^2+\omega^2}$
8	$cos(\omega t)u(t)$	$\frac{p}{p^2+\omega^2}$
9	$e^{-at}Sin(\omega t)u(t)$	$\frac{\omega}{(p+a)^2+\omega^2}$
10	$e^{-at}cos(\omega t)u(t)$	$\frac{p+a}{(p+a)^2+\omega^2}$
11	$(Sin(\omega t)+ \omega t\, cos(\omega t))u(t)$	$\frac{2\omega p^2}{(p^2+\omega^2)^2}$
12	$(cos(\omega t)+ \omega t\, sin(\omega t))u(t)$	$\frac{p(p^2+3\omega^2)}{(p^2+\omega^2)^2}$
13	$e^{-at}Sinh(\omega t)u(t)$	$\frac{\omega}{(p+a)^2-\omega^2}$
14	$e^{-at}cosh(\omega t)u(t)$	$\frac{p+a}{(p+a)^2-\omega^2}$

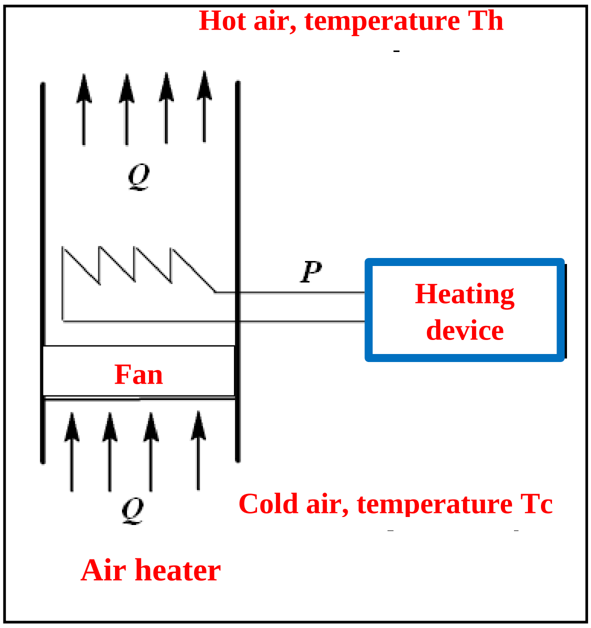
Exercise Series -1-

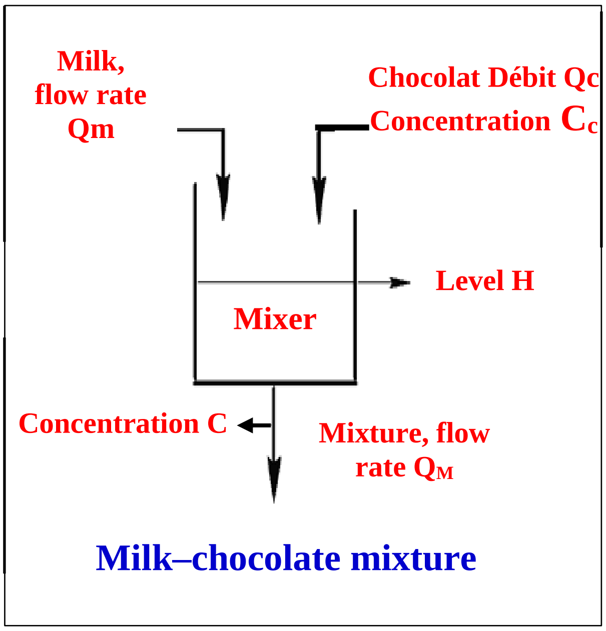
Fundamentals of Control Systems

Exercise 1:

Determine the functional block diagram for the following systems:

- **Air Heater:** An air flow rate Q is heated to a temperature T by means of an electric resistor supplied with an electrical power P. A two-speed fan enables the desired air flow rate to be achieved. The variable to be controlled is the temperature T_c. The variables influencing T_c are the air flow rate Q, the inlet air temperature Tc, and the electrical power P.
- **Milk and Chocolate Mixer:** To industrially produce chocolate milk, chocolate is mixed with milk. The goal is to control the quality of the resulting mixture based on an analysis that provides the chocolate concentration. The level indicator is used to determine the volume in the mixer. The variables to be controlled are the level H and the concentration C. The variables influencing the level H are the milk flow rate Qm the chocolate flow rate Q_C, and the mixture outflow rate Q_M. The variables influencing the mixture concentration C are the milk flow rate Qm, the chocolate flow rate Q_C, and the chocolate concentration C_C.

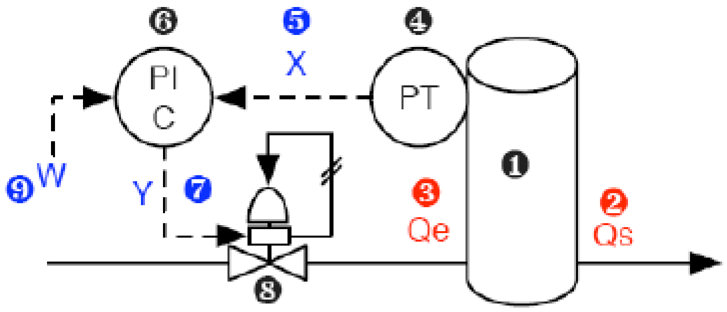




Exercise 2:

Consider a pressure control system for a tank containing a solvent, as illustrated in the diagram below:

- **PIC** : Pressure Controller
- **PT** : Pressure transmitter
- **Qe**: Inlet pressure flow into the tank
- **Qs**: Disturbance
- **8**: Control valve (pressure reducing valve)



Determine the functional block diagram of this control loop.

Exercise 3:

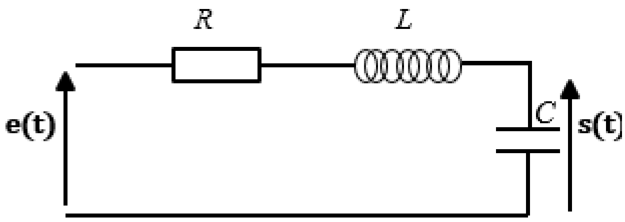
Automatic Bread Toaster

Open-loop operation:
This is a control system governed by a timer. The time required to obtain "properly toasted bread" must be estimated by the user, who is not part of the system itself.
Closed-loop operation:
Assume that the heating element provides the same amount of heat on both sides of the bread, and that the toasting quality is determined by the bread color. The system is initially calibrated by means of an adjustment screw to set the desired toasting level (setpoint). When the switch is closed, the bread is toasted until its color (detected by a color sensor) becomes close to the desired color. At this point, the switch opens automatically through a feedback mechanism, which may be mechanical or electrical.
Provide the functional block diagram for both cases.

Exercise Series -2-

Laplace Transforms and Representation of Feedback Control Systems

Exercise 1: Consider the following RLC circuit. Determine the current *i(t)* by applying the Laplace transform.



Electrical circuit: RLC

Exercise 2:

Determine the Laplace transforms of the following functions:

1- $f_1(t) = 8te^{3t}\, U(t)$
2- $f_2(t) = (\ e^{-9t}(7t^2 - 5)\ )\, U(t)$.
3- $f_3(t) = (2e^{-4t}(\cos 3t + \sin 3t))\, U(t)$.
4- $f_4(t) = (\sin(5t + 30^0))\, u(t)$

Exercise 3:

Give the Laplace transform of the function *f*:

$$f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2 - t & 1 \leq t \leq 2 \end{cases}$$

Exercise 4:

Find the Laplace transform of the function:

$$f(t) = 2e^{-t}\cos 10t - t^4 + 6\, e^{-(t-10)}\ u(t - 10).$$

Exercise 5:

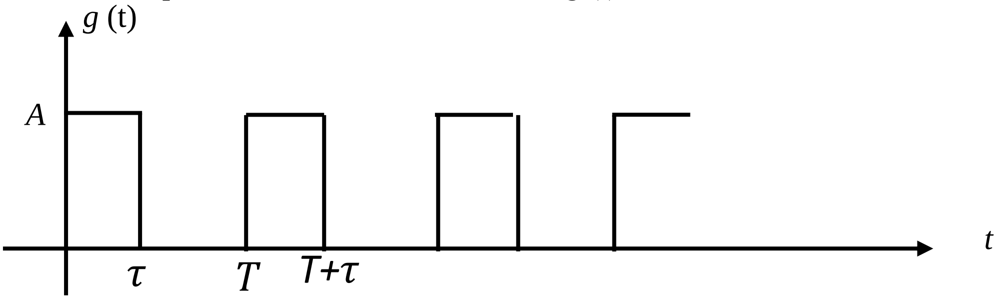
Find the inverse Laplace transforms of the following functions:

1- $F_1(P) = \frac{(p+1)}{(p^2+3p+2)}$
2- $F_2(P) = \left(\frac{5!}{(p-2)^2}\right)$
3- $F_3(P) = \left(\frac{3(p+1)-2}{(p+1)^2+4}\right)$
4- $F_4(P) = \left(\frac{2}{(p+2)(p^2+4p+4)}\right)$

Exercise 6:

1- Calculate the Laplace transform of the function *f* (t):

2- Deduce the Laplace transform of the function *g (t).*



Exercise 7:

Using the initial and final value theorems, calculate: *f* (0), *f* (∞) for the following functions

1- $F_1(P) = \frac{p^2+2p+4}{p^3+3p^2+2p}$

2- $F_2(P) = \frac{p^3+2p^2+6p+8}{p^3+4p}$

Exercise 8:

Use the Laplace transform to solve the following differential equations, and then determine their general solution by applying the inverse Laplace transform.

1) $y'(t) + 2\,y(t) = e^{-t}$ ***avec***: $x(0) = 2$

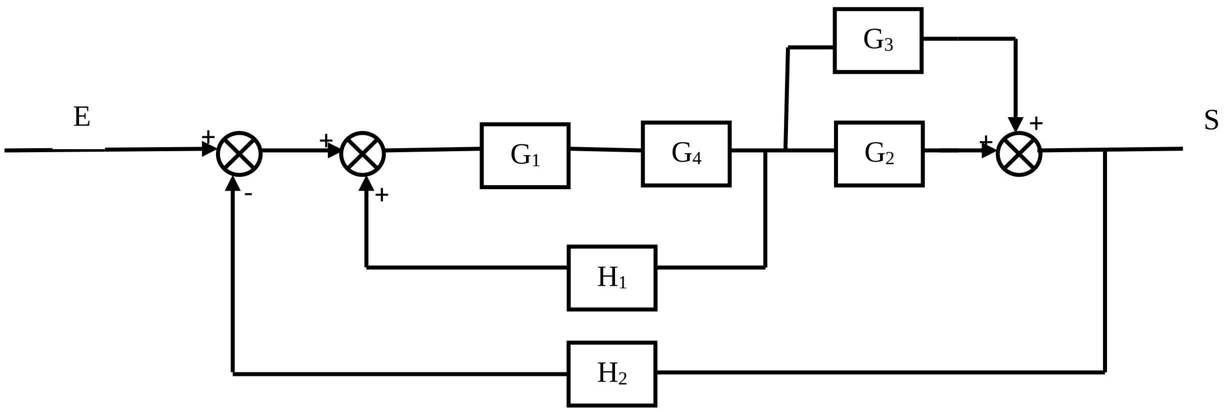
2) $y'(t) = 2y(t) + 2t\,e^{2t}$ ***avec*** : $y(0) = 1$

3) $x''(t) + 6\,x'(t) + 9\,x(t) = e^{-2t}$ ***with***: $x(0) = 0$ ***et*** $x'(0) = 0$

4) $y''(x) - \frac{5}{2}y'(x) + y(x) = -\frac{5}{2}\sin x$ ***with***: $y(0) = 0$ ***et*** $y'(0) = 2$

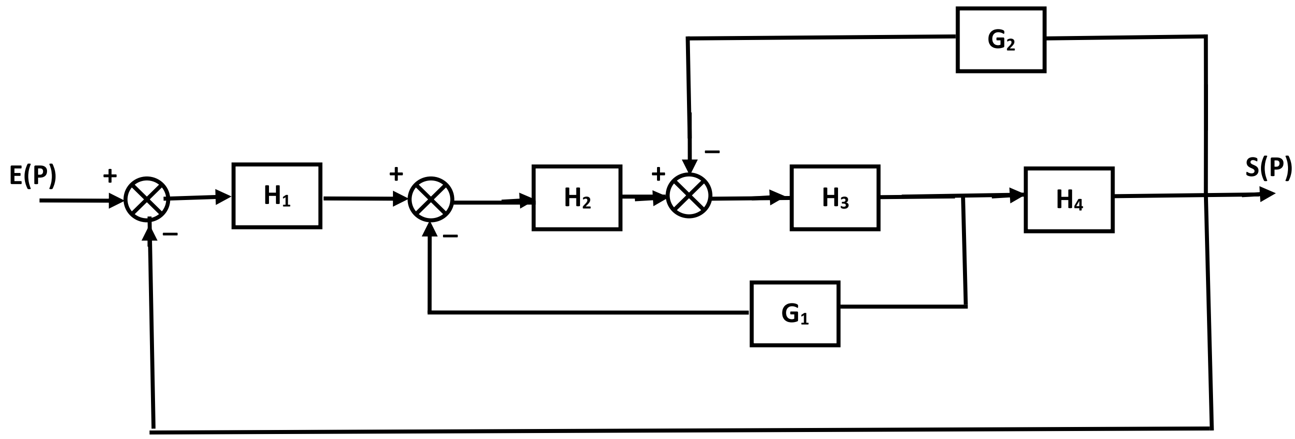
Exercise 9:

Determine the transfer function by successive simplification of the functional blocks:



Exercise 10:

Consider the following block diagram. Simplify it and determine the transfer function F(p).

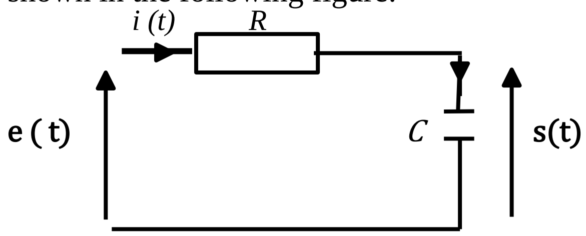


Exercise Series -3-

Time-Domain Analysis

Exercise 1:

Consider the RC circuit shown in the following figure:



The injected input signal is *e (t)* = 3t.
Determine the expression of *s (t)*.

Exercise 2:

Consider the system with the following transfer function:

$$G(p) = \frac{0.5(1-p)}{(1+p)(1+0.5p)}$$

1. Express the system as two first-order systems.
2. Determine and plot the poles and zeros in the complex plane.
3. A unit step input *u(t)* is applied to the system :
4. Determine the expression of y (t); and evaluate y(0) and y(t) as t→∞ .
5. Study the variation of y (t) and graphically represent the evolution of the output.

Exercise 3:

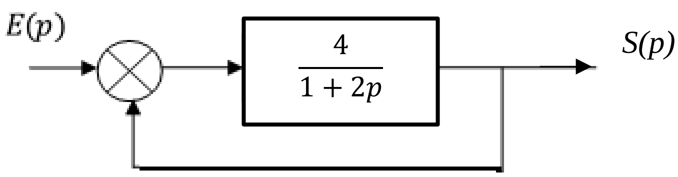
Calculate the step response and the impulse response of the two systems defined by the following transfer functions:

$$G_1(p) = \frac{2(p+1)}{p(p+3)^2}$$

$$G_2(p) = \frac{(p+4)}{(p+1)(p^2-4p+4)}$$

Exercise 4:

Consider the following unity-feedback control system:

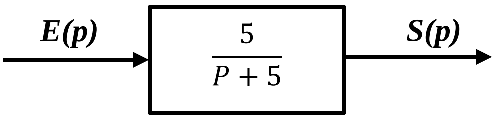


1- Determine the order of the system, and explain the significance of the parameters 4 and 2.

2- Determine the transfer functions in open loop (OL) and closed loop (CL).

Exercise 5:

Consider the following first-order control system:



1. Calculate and plot the step response *s (t)* for a unit step input *e (t) =1(t).*
2. Determine the response time.

Exercise 6:

Consider a second-order system governed by the equation:

$$\ddot{y} + 0.4y + 0.2\dot{5}y = e.$$

1- Write the transfer function in standard form.

1. Determine the value of *K*, the natural frequency f_0, and the damping ratio ξ.

Exercise 7:

Consider a system with the following transfer function:

$$G(\mathrm{P}) = \frac{-P + 5}{p^2 + 5p + 4}$$

Calculate the time-domain response *y (t)* of the system for the input *e (t)* as follows:

1. A Dirac impulse.
2. A unit step.

Exercise 8:

For each of the following first-order systems, calculate the step responses, then determine the rise time t_r and the settling time t_s in 5%:

$$G_1(\mathrm{p}) = \frac{5}{p+5} \quad , \quad G_2(\mathrm{p}) = \frac{20}{p+20}$$

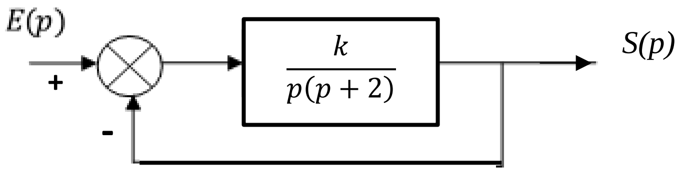
Exercise 9:

For each of the following second-order systems, calculate , ω_0 , t_p , t_r ,t_s , in *5%*:and *d%* :

$$G_1(\mathrm{P}) = \frac{120}{p^2+12p+120} \quad , \quad G_2(\mathrm{p}) = \frac{0.01}{p^2+0.002p+0.01}$$

Exercise 10:

Consider the following unity-feedback system:



Calculate the gain K in order to ensure a 10% overshoot (*d%* ≤10%) on the step response.

Exercise 11:

Consider the system with the following transfer function:

$$G(\mathrm{P}) = \frac{100}{p^2 + 15p + 100}$$

Find: t_p, D,t_r and t_s

Exercise Series -4-

Frequency Domain Analysis of Systems

Exercise 1:

Plot the Bode diagram, showing both the gain and phase curves, for the following transfer functions:

$$F_1(\mathrm{P}) = \frac{1}{p+100} \quad , \qquad F_2(\mathrm{P}) = \frac{1000}{(p+1)(p+100)} \quad , F_3(\mathrm{P}) = \frac{220}{p^2+13p+22}$$

Exercise 2:

Consider the system described by the following transfer function:

$$G\mathrm{P}) = \frac{100}{(1+0.2p)(1+0.4p)(1+2.5p)}$$

Plot the two Bode diagrams: magnitude (gain) and phase.

Exercise 3:

Plot the Bode and Nyquist diagrams of the following function:

$$T(\mathrm{P}) = \frac{5P}{1+5p}$$

Exercise 4:

Plot in the Nyquist plane the open-loop transfer function *T* (p) of a unity-feedback system with forward-path transfer function G(p).

$$T(\mathrm{P}) = \frac{4}{\tau P - 1} \quad ; \quad \tau > 0.$$

Exercise 5:

Plot the Black and Nyquist diagrams of the following systems:

- $F(\mathrm{P}) = \frac{5}{1+2P}$
- $G\mathrm{P}) = \frac{5}{(1+p)(1+2p)}$

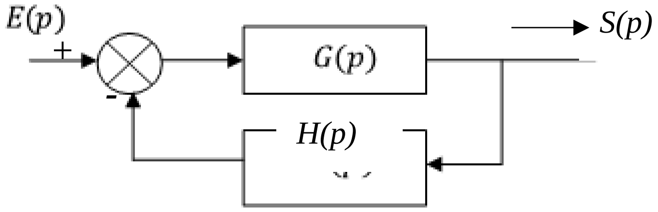
Exercise 6:

Determine the stability condition of the system defined by its open-loop transfer function *F* (p).

$$F\text{P}) = \frac{2K}{P(1+\ \tau\, p)}$$

Exercise 7:

Examine the stability of the following system in both open-loop and closed-loop configurations.



With: $G(\text{P}) = \frac{10}{3p^3+2p^2+p+1}$ and *H(p)= 1.9 p+1*

Exercise 8:

Consider the following system:

$$G\ (p)= \frac{10}{p^5+2p^4+\ 3p^3+\ 6p^2+\ 5p+3}$$

Study its stability using the Routh criterion.

Exercise 9:

Is the system defined by:

$$1+T(p) = p^5 + p^4 +\ 3p^3 +\ 4p^2 + p+2$$

Stable ?

Exercise 10:

Study the stability of the following system:

$$1+T(p) = 4p^5 + 10p^4 +\ 10p^3 +\ 20p^2 + p + 1 = 0$$

Exercise 11:

1. Using the Nyquist criterion, determine the stability condition of the following system:

$$T(j\omega) = \frac{k}{j\omega(1+Tj\omega)(1+aTj\omega)} \quad with\!: k > 0\ \ , a > 0\ \ and \quad T > 0.$$

2. Examine the stability of the system with the following transfer function using the Nyquist criterion:

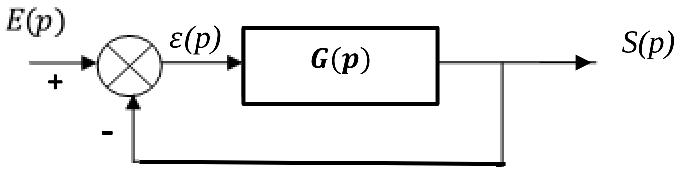
$$G(\text{P}) = \frac{4}{\tau\, p - 1} \quad ; \qquad \tau > 0.$$

Exercise Series -5-

Synthèse des systèmes

Exercise 1 :

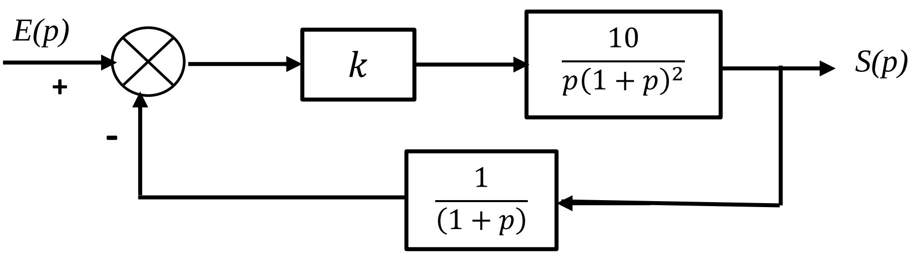
Consider a unity-feedback system defined by:



- With: GP) $= \frac{k}{(5p+2)(p+3)^2}$ in the forward path.

Determine the range of values for k such that the system remains stable and the steady-state error for a unit step input *e (t) = 1(t)* does not exceed 10%.

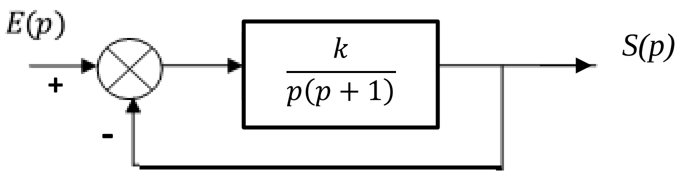
Exercise 2 :



Find the limits for the gain k to ensure that the system is stable and that the steady-state error for a unit step input e(t) =1 (t) and for a unit ramp input remains below 10%.

Exercise 3 :

For the system shown in the figure below, plot the root locus of the closed-loop transfer function (CLTF), taking *K* as the tuning parameter.



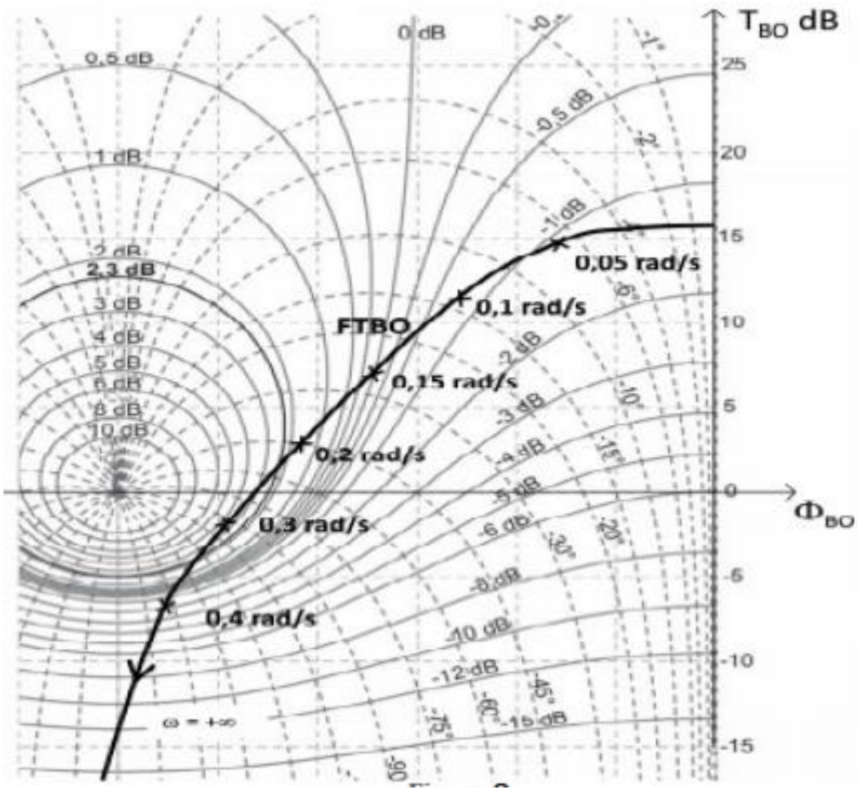
Exercise 4:

Determine the value of *K* that guarantees a percentage overshoot d(%)<6%, given that:

$$F(p) = \frac{K}{8p^3 + 21p^2 + 10.5p + K + 1}$$

<u>Exercise 5:</u>

Consider a unity-feedback system whose open-loop transfer function (OLTF) is represented in the Black plot shown in the figure below. The curve is marked with six frequency points (unit: rad/s).



Determine the gain margin and phase margin of the considered system. Conclude on the closed-loop stability.

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